Refinements in Continuous Galerkin Wetting and Drying Algorithms

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Abstract

Coastal ocean hydrodynamic models are used to simulate water surface elevations and circulation in oceans, lakes, estuaries, rivers and floodplains. One such model is ADCIRC (ADvanced CIRCulation), which was originally developed more than 15 years ago and which is used for a variety of purposes, including naval fleet operations, storm surge predictions, and larvae transport. ADCIRC assumed fixed land boundaries until a wetting and drying algorithm was implemented in 1995. However, this algorithm had two components that limited the performance of the model. First, nodes were required to remain "wet" or "dry" for a user-specified number of time steps before changing states. This component became restrictive in relatively flat regions, such as a flood plain, where it caused oscillations and slowed the propagation of flood waves. Second, in regions with steep bathymetry, mass balance problems and instabilities would occur when a thin film of water was allowed to flow uninterrupted. Changes based on a more physically-accurate description of the wetting and drying process were made recently to address these two problems. This paper describes the wetting and drying algorithm and those changes, and it applies the improved algorithm to an idealized domain that was designed specifically to test the two problem areas. The improved algorithm provides better stability and mass balance properties.

1. Background

Shallow water equations are used by researchers and engineers to model the circulation of oceans, coastal areas, estuaries, lakes and impoundments (Kolar et al. 1994a). The finite element solutions of these equations have been improved by two related equations: the wave continuity equation (WCE), introduced by Lynch and Gray (1979) to suppress the spurious oscillations inherent to the primitive equations without having to dampen the solution either numerically or artificially; and the generalized wave continuity equation (GWCE), introduced by Kinnmark (1986) to allow a balance between the primitive and pure wave forms of the shallow water equations by using a

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weighting parameter *G*. The finite element model used in this paper, ADCIRC, was developed from the GWCE (Luettich and Westerink 2005).

One process with which ADCIRC and other hydrodynamic models may have accuracy and mass balance problems is wetting and drying. Large-scale water behavior is often driven by wind and tides; for the latter, the most notable is the M_2 tide caused by the gravitational effects of the moon. Where the tides meet a sloping shoreline, the water should move up and down the beach, causing areas to alternate between being wet and dry. Simply ignoring this behavior and treating the shoreline as a firm boundary, as was done in early versions of ADCIRC, allows the water to build up on boundary nodes as if against a vertical wall. Not only is this method qualitatively incorrect, it also affects the manner in which waves reflect.

To simulate wetting and drying in a numerical model, researchers have employed several methods. Some models do not fix the grid in space, so that the wet/dry interface is allowed to advance and recede naturally (Lin et al. 2004). However, this approach can produce contorted elements in regions with complex geometries and/or large floodplains. Some models allow elements to wet and dry gradually, so that an element could be half-wet and half-dry (Bradford and Sanders 2002). This approach does allow for a fixed grid, but it requires extensive changes to the current computational algorithm.

The ADCIRC wetting and drying algorithm turns on and off elements as they are wetted and dried. The algorithm was developed by Luettich and Westerink (1995a, 1995b) and is based on simplified physics and some empirical rules. In general terms, the algorithm uses a minimum depth to dry nodes and a simple momentum balance to wet nodes. As nodes are wetted and dried, they are included and excluded from the calculations, respectively, so that the problem size can change during each time step. This algorithm was implemented in the two-dimensional (*x-y*) version of ADCIRC and proved adequate in many problems.

However, the wetting and drying algorithm lacked robustness for the most difficult of problems, including relatively flat regions, such as flood plains. To address these problems, the wetting and drying algorithm was updated in 2004. Two parameters were removed entirely, and the algorithm itself was lengthened. As we will describe in Section 2.2, these updates were implemented to address specific situations where the original wetting and drying algorithm struggled.

The primary objective of this paper is to verify that these changes were necessary and beneficial. We describe the algorithm and the changes, and we apply the "original" and "improved" algorithms to an idealized domain that was designed specifically to test the problem areas. In so doing, we also accomplish two secondary objectives: (1) to test the wetting and drying algorithm in the three-dimensional ADCIRC model, in which it was only recently implemented; and (2) to quantify the behavior of the algorithm under a range of input parameters. The latter objective is particularly important for model users, who may not understand how these parameters affect the behavior of the model in specialized situations.

In Section 2, we describe the wetting and drying algorithm, the changes made, the model problem used in this study, and the error measure we used. In Section 3, we present the results from a set of numerical experiments using the two-dimensional (*x-z*) ADCIRC model, and, in Section 4, we present the results from a similar set of numerical experiments using the three-dimensional ADCIRC model. Finally, in Section 5, we draw conclusions based on the results of these experiments.

2. Methods

This section contains a description of the original wetting and drying algorithm, a synopsis of the two major changes, a description of the model problems used in this study, and a review of the error measure used in our numerical experiments.

2.1. Wetting and Drying Algorithm

The one-dimensional ADCIRC wetting and drying algorithm is an approach developed by Luettich and Westerink (1995a, 1995b) and is based on simplified physics and some empirical rules. The algorithm occurs in the middle of the time step, between the solution of the GWC equation (for water surface elevations) and the solution of the momentum equation (for velocities or fluxes). The original algorithm is comprised of three parts.

First, the total water depth at every node is checked against a minimum wetness height, H_{min}. If the total water depth is larger than this minimum value, then the node remains active ("wet") and is included in the rest of the calculations. However, if the total water depth has fallen below this minimum value, then the node is deemed inactive ("dry") and removed from the calculations. Note that a dry node can have a positive water depth that is smaller than H_{min}. To help control oscillations, an input parameter $N_{wet\text{-}min}$ allows the user to control the number of time steps that a node has to remain wet before it can be turned off. However, as we will discuss in Section 2.2, this parameter was removed as part of the updates to the algorithm.

Second, the steady state velocity that would result from a balance between the water level gradient and the bottom friction between a wet and an inactive node is checked against a minimum wetting velocity, U_{min}. The balance is given by:

$$U = \frac{g(\zeta_{i-1} - \zeta_i)}{\tau_i \Delta x_i}, \tag{1}$$

where g is gravity; ζ_{i-1} and ζ_i are the free surface elevations at the adjacent node and the node of interest, respectively; τ_i is the equivalent linear bottom friction coefficient, given by:

$$\tau_i = C_{f-\tau}\left(\frac{|u_i|}{H_i}\right), \tag{2}$$

where $C_{f-\tau}$ is a bottom friction coefficient that is specified in the input file; and Δx_i is the grid spacing. Note that in many situations, only the free surface elevations will change significantly from time step to time step. In this case, the U_{min} criterion almost becomes a height restriction, where a node becomes active if the adjacent node's free surface elevation is sufficiently larger than its own. Again, to help control oscillations, another input parameter $N_{dry\text{-}min}$ allows the user to control the number of time steps that a node has to remain inactive before it can be wetted. However, this parameter was also removed as part of the updates to the algorithm.

Third, every landlocked wet node is tagged as inactive. A landlocked wet node is not connected to any active elements, and thus does not receive contributions to either side of the equation corresponding to that node. For some bathymetries, this criterion allows a node to remain inactive even if its total water depth is larger than the minimum wetting height.

2.2. Updates to the Wetting and Drying Algorithm

In the summer of 2004, two changes were made to the wetting and drying algorithm. First, the two parameters $N_{dry\text{-}min}$ and $N_{wet\text{-}min}$ were eliminated. Second, an elemental drying check was added.

The first change relates to the propagation of waves on relatively flat flood plains. The user specified the two parameters $N_{dry\text{-}min}$ and $N_{wet\text{-}min}$ in the input file, and they were used to control how long a node had to remain either dry or wet. Thus, if $N_{dry\text{-}min}$ was set to 20, then any node had to remain dry for at least 20 time steps. These parameters were included originally to control oscillations at the wetting front, but it was discovered that they also slowed the propagation of flood waves. To prevent this, the two parameters were removed from the algorithm.

The second change relates to the flow of water down a steep incline. It was discovered that the simple momentum balance used in the node-based wetting check allowed receiving nodes of elements in areas with steep topography to remain active even when they were "barely wet." Thus, a thin film of water would be allowed to remain wet if it was on an incline where water was flowing from above. Mass balance problems occurred in these regions, because the model requires water depths to remain above a value of $0.8H_{min}$. (If the water depth falls below this value, then it is artificially lifted up, and thus mass is added to the system.) To prevent this, an elemental drying check was added to the algorithm. A new parameter, H_{off}, was hardwired into the code, and it is set to 120 percent of the H_{min} parameter. (The "120 percent" criterion was an *ad hoc* selection; nonetheless, it works.) If the receiving node on an element has a water depth that is less than H_{off}, then the element itself is dried. This change forces water to build up on an incline before it is allowed to flow.

In this study, we test the relative behavior of the "original" and "improved" wetting and drying algorithms. For the numerical experiments using the two-dimensional (*x-z*) ADCIRC model, we will present results for both versions of the wetting and drying algorithm, so that their behavior can be quantified. For the numerical experiments using the three-dimensional ADCIRC model, we will present results from only the improved algorithm. We expect the improved algorithm to show significantly better results in model problems that contain the features for which they were designed.

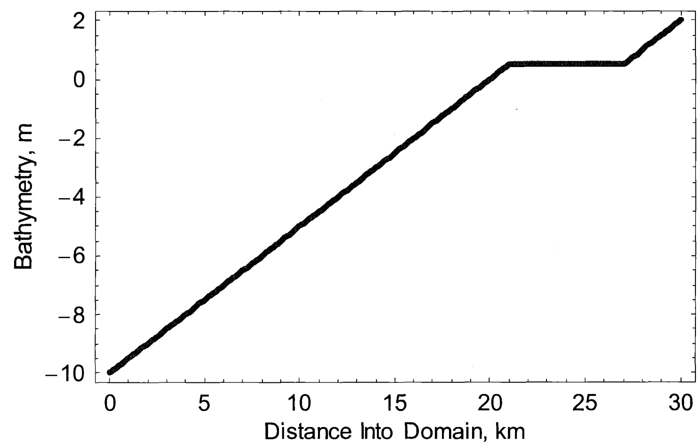

Figure 1. A schematic of the bathymetry for the Plateau-2D domain. The flat region has a bathymetry of 0.5 meters (above sea level) and extends between the x-distances of 21 kilometers to 27 kilometers.

2.3. Model Problems

In the interest of brevity, we will apply each version of the ADCIRC model to only one model problem. This model problem was designed to test the updates to the wetting and drying algorithm and to define an optimal set of input parameters for simulations of this type. However, beside the extra dimension in the three-dimensional model problem, there are other subtle differences between the two-dimensional (x-z) and three-dimensional model problems. These differences are described below.

The two-dimensional (x-z) Plateau-2D domain is shown in Figure 1. It was designed to test the improved wetting and drying algorithm. Specifically, there are two features that distinguish it from the classic linear sloping beach domain, which is often used to test wetting and drying algorithms (Carrier and Greenspan 1958, Siden and Lynch 1988). First, the Plateau-2D domain has a 6-kilometer region where the bathymetry is a constant topography of 0.5 meters above sea level. This feature should test the improved algorithm's ability to simulate flood waves, which tended to be slowed down by the $N_{dry\text{-}min}$ and $N_{wet\text{-}min}$ parameters in the original algorithm. The forcing amplitude is 1 meter at the open ocean boundary, so the waves should wet the sloped beach and then flow across the flat region. Second, the Plateau-2D domain has sloped regions that are relatively steep. This feature should test the improved algorithm's new elemental drying check, which was added to better simulate flow down steep inclines. As the tide recedes, water from the flat region should drain down the incline. In addition to those two features, this problem has the following parameters (unless stated otherwise): a total length of 30 kilometers, an undisturbed water length of 20 kilometers, a bathymetric depth at the open ocean boundary of 10

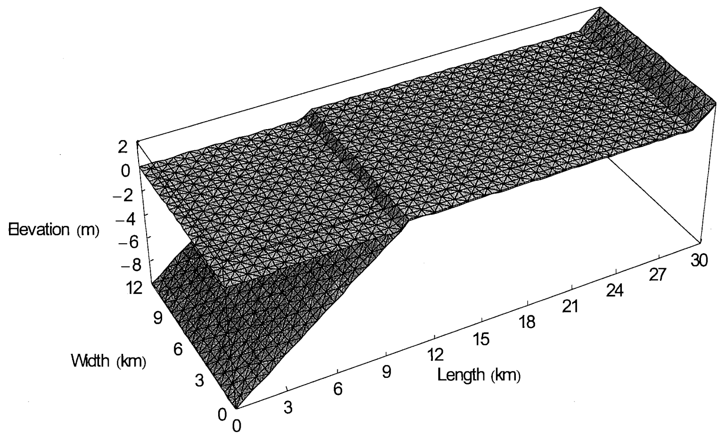

Figure 2. Three-dimensional view of the Plateau-3D domain. Note that the flat range has a bathymetry of 0.5 meters above sea level, and it extends from 10.5 kilometers to 28.5 kilometers in the x-direction.

meters, a grid spacing of 250 meters, 11 vertical layers, a time step of 1 second, a forcing amplitude of 1.0 meter, a tidal period of 12 hours (43,200 sec), a duration of 4 tidal periods, a linear bottom stress coefficient (defined in Section 3.2) of 0.0001 and a G value of 0.01 sec^{-1}, an H_{min} value of 0.01 meters, and a U_{min} value of 0.01 meters per second.

The three-dimensional Plateau-3D model problem was also designed explicitly to test the updates in the wetting and drying algorithm. It is similar to the Plateau-2D domain, except it has been extended in the y-direction for a distance of 12 kilometers. A schematic of this domain is shown in Figure 2. There are three significant differences from the Plateau-2D domain. First, the grid spacing has been increased from 250 meters to 500 meters, in order to keep the model stable at the same time step of 1 second. Second, the flat region of the domain has been tripled in length, from 6 kilometers to 18 kilometers, to create a difficult robust study of the wetting and drying of floodplains. Third, the slope at the left and right edges of the domain has been doubled, to create a more difficult study of drainage down a steep incline. Taken together, these changes make simulations on the Plateau-3D domain even more difficult, and it should be a challenging test of the improved wetting and drying algorithm. This problem has the following parameters (unless stated otherwise): an total length of 30 kilometers, an undisturbed water length of 10 kilometers, a bathymetric depth at the open ocean boundary of 10 meters, a grid spacing of 500 meters, a time step of 1 second, a forcing amplitude of 1.0 meter, a tidal period of 12 hours (43,200 sec), a duration of 4 tidal periods, a linear slip coefficient of 0.0001 and a G value of 0.01 sec^{-1}, an H_{min} value of 0.01 meters, and a U_{min} value of 0.01 meters per second.

2.4. Error Computations

In the absence of an analytic solution, our practical error measure is an examination of mass balance. For the studies of the two-dimensional (*x-z*) ADCIRC model, mass balance errors are calculated using a finite volume approach, where we compute the difference between the global accumulation and the global mass flux, as represented by the vertically-averaged primitive continuity equation. We utilize a cumulative mass balance error over the entire simulation.

For the studies of the three-dimensional ADCIRC model, instead of using a depth-averaged flux and computing the mass balance error at each horizontal element, we now compute mass balance error at all elements. We begin with the three-dimensional continuity equation: $\nabla \cdot \vec{v} = 0$, where $\vec{v}$ is the three-dimensional velocity. If we integrate this equation over some region Ω (here taken to be an individual element), we get:

$$\int_{\Omega} (\nabla \cdot \vec{v}) d\Omega = 0 . \tag{3}$$

We can then apply the divergence theorem on an individual element to get:

$$\int_{\partial\Omega} (\vec{v} \cdot \vec{n}) d(\partial\Omega) A = 0 , \tag{4}$$

where $\vec{n}$ is the unit normal vector on each face of the element. When we compute the fluxes on an element, we first compute an average velocity on each face as an arithmetic average of the velocities at the three (or four) nodes that define the face. (This amounts to a one-point Gaussian quadrature, which is exact for a linear function.) Then, to get the flux, we take the dot product of that average velocity and the unit normal vector. Thus, the local mass balance error for any element is simply the residual when that element's fluxes are summed. The global mass balance error is a sum of those residuals. To allow for a comparison with the study of the two-dimensional (*x-z*) ADCIRC model, we divided the global mass balance error by the width of the domain to obtain an error per unit width.

As we will show, this new method of computing mass balance errors prevents them from being truly comparable to the errors reported in the one-dimensional and two-dimensional (*x-z*) studies. When the three-dimensional ADCIRC model solves for vertical velocities, it uses the same equation that we are using to solve for fluxes for mass balance. In effect, the vertical velocities are computed in a way that minimizes mass balance error. Because of the continuous finite element approximation, the vertical velocity solver couples the entire water column, so, while the "global" mass balance errors are minimized for the column, the local mass balance

Table 1: Summary of heuristic stability results for the two versions of the wetting and drying algorithm. The two versions show significant differences for the Plateau-2D domain. Note that mass balance error is an average error over all four tidal cycles.

	Original Algorithm	Improved Algorithm
Maximum stable time step	10 sec	20 sec
Mass balance error, at the maximum stable time step	15881 m^2	20148 m^2
Mass balance error, at a time step of 1 second	NA	5662 m^2

error at any element in the column may be non-zero. We will show that this method of computing mass balance produces much smaller errors, because it is more closely tied to the way the model solves for vertical velocities.

Recently, several papers (Hughes et al. 2000, Berger and Howington 2002) have advocated computing mass balance from finite element residuals in order to be consistent with the numerical discretization. However, we have shown (Kolar et al. 1994b) the finite volume approach to be a good surrogate variable for accuracy and phasing errors; that is, small mass balance errors (as computed with finite volume) correlate with small constituent errors. Additionally, we have shown that the finite volume method is a good indicator of truncation errors, especially for domains that have a constant node spacing (Dietrich et al. 2005). These results suggest that the mass checker is a good error assessment tool for wet/dry simulations.

3. Numerical Experiments, 2D (*x-z*) ADCIRC

This subsection contains the results of numerical experiments conducted using the two-dimensional (*x-z*) ADCIRC model and the Plateau-2D domain. As discussed in Section 2.3, the Plateau-2D domain was designed to test the behavior of the improved wetting and drying algorithm. The domain features a flat region to simulate tidal waves on a flood plain, and it also features a steeper slope to simulate thin films of water draining downhill. This subsection contains the results of five numerical experiments: heuristic stability, parameter sensitivity with G and K_{slip}, parameter sensitivity with H_{min} and U_{min}, horizontal resolution, and vertical resolution.

3.1. Heuristic Stability

Table 1 summarizes the heuristic stability results for the Plateau-2D domain and the two versions of the wetting and drying algorithm. These results show a significant difference between the two versions of the algorithm. The improved algorithm increases the maximum stable time step by 100 percent, while at the same time increasing the mass balance error by only about 27 percent. Most importantly, the original algorithm does not converge as the time step is reduced; in fact, the model becomes unstable at a time step of 1 second. We will examine this behavior by holding the time step at 1 second for the rest of the Plateau-2D domain results.

3.2. Parameter Sensitivity - K_{slip} and G

Two important parameters in the ADCIRC model are the bottom friction and the numerical parameter G (sometimes called τ_0). In two (x-z) dimensions, bottom friction is implemented as a term in the vertical stress calculation:

$$\frac{\tau_{bx,j}}{\rho_0} = K_{slip,j} u_j, \tag{5}$$

where τ_{bx} is the bottom stress; ρ_0 is the reference density of water; u is the velocity; j is the node index; and K_{slip} is the bottom boundary condition. For a "no slip" bottom boundary condition, the velocity is simply set to zero as part of the solution process. For a "linear slip" bottom boundary condition: $K_{slip,j} = \text{constant}$. And, for a "quadratic slip" bottom boundary condition: $K_{slip,j} = C_d|u_j|$, where C_d is a quadratic drag coefficient.

The two-dimensional (x-z) ADCIRC model does not include all three types of bottom boundary condition; only the linear slip bottom boundary condition is available. The results shown in this section utilize that linear slip bottom boundary condition, where K_{slip} is a user-specified constant. We varied both K_{slip} and G from 10^{-5} to 10^{0} (sec^{-1} for G, unitless for K_{slip}), creating a test matrix of 36 combinations of K_{slip} and G. Then we examined the effect of each combination on the model's accuracy and mass balance properties, for both the original and improved versions of the wetting and drying algorithm.

Figure 3 shows the mass balance errors for the original wetting and drying algorithm, and Figure 4 shows a similar graph for the improved algorithm. As noted in Section 2.4, these mass balance errors reflect an average error over all four tidal cycles. The improved algorithm is unstable for fewer combinations of G and K_{slip} than the original algorithm. And, when both versions of the algorithm are stable, the improved algorithm produces significantly smaller mass balance errors.

In a previous study of the original wetting and drying algorithm and a one-dimensional version of the ADCIRC model, it was found that the optimal combination was $G = 0.01$ sec^{-1} and $K_{slip} = 0.0001$ (Dietrich et al. 2004). These results do not disprove that is the optimal combination. The mass balance error is larger at that combination than it is when the parameters are increased, but we saw similar behavior for the linear sloping beach domain, where this combination proved to be the most accurate. Thus, we will continue to use these values for G and K_{slip}, even though the original wetting and drying algorithm is unstable at that combination. Of course, although model users are free to select any value for the numerical parameter G, they must select a value for K_{slip} that is both stable and physically realistic. These results

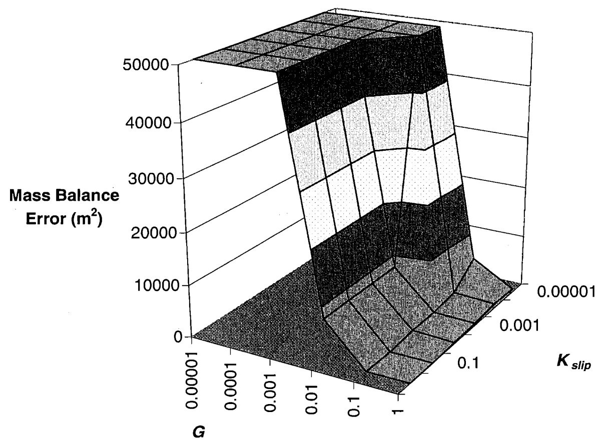

Figure 3. Mass balance errors for the original wetting and drying algorithm applied to the Plateau-2D domain, for a range of G-K_{slip} combinations. Note that we have cropped the vertical scale for comparison to Figure 4; some combinations where G is small and K_{slip} is large are stable, but they produce undesirably large mass balance errors.

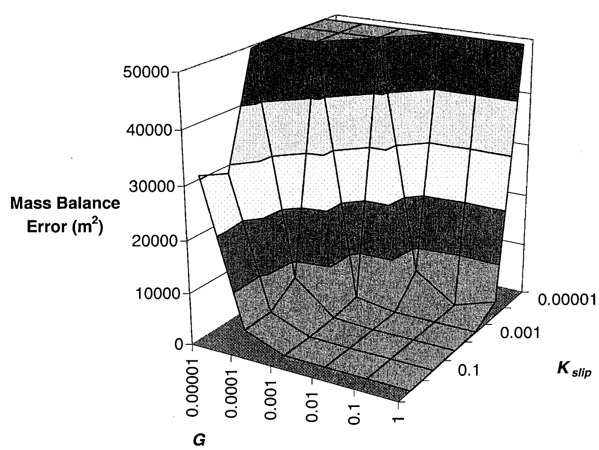

Figure 4. Mass balance errors for the improved wetting and drying algorithm applied to the Plateau-2D domain, for a range of G-K_{slip} combinations.

and a previous study using a linear sloping beach domain suggest that K_{slip} should be assigned a value in the range near $K_{slip} = 0.0001$.

3.3. Parameter Sensitivity - H_{min} and U_{min}

The effect of the wetting and drying parameters H_{min} and U_{min} (defined in Section 2.1) was examined by varying each parameter from 10^{-5} to 10^{0} (meters for H_{min}; m/sec for U_{min}), creating a matrix of 36 combinations. Figure 5 shows the mass balance errors produced by the original wetting and drying algorithm, while Figure 6 shows a similar graph for the improved algorithm. As noted in Section 2.4, these mass balance errors reflect an average error over all four tidal cycles. Note that our selection of $G = 0.01$ sec^{-1} and $K_{slip} = 0.0001$ influence our results; the original algorithm is unstable for most combinations of H_{min} and U_{min}, and it produces large errors when it is stable.

The improved algorithm, on the other hand, is stable and produces reasonable errors for all combinations. Note that its smallest errors occur when H_{min} is largest and thus most restrictive; at $H_{min} = 1.0$ meters, the model is prevented from wetting the flat region of the Plateau-2D domain, and thus the mass balance errors are much better. The errors are uniform as H_{min} is decreased, meaning that model users can use any value of that parameter that is sufficiently small. And the U_{min} parameter does not have a noticeable effect on model behavior.

3.4. Horizontal Resolution

Figure 7 shows the mass balance errors for a range of horizontal resolutions. The mass balance errors do not converge as the resolution is refined; in fact, many of the largest errors occur when the grid spacings are relatively small. This behavior is a concern, because a good algorithm should converge as the resolution is refined. We hypothesize that the Plateau-2D domain is not a good test of horizontal resolution because, at fine resolutions, more nodes are placed at the top of the steep incline, where the drainage from the floodplain causes elements to oscillate between being wet and dry. Because ADCIRC's wetting and drying algorithm is based on adding and subtracting elements (and thus the addition and subtraction of large volumes of water) to the computational domain, the mass balance errors occur primarily when elements are wetted and dried. These processes occur more often at fine resolutions that have more elements overall. This is a consequence of the ADCIRC wetting and drying paradigm, but it can be minimized. For the Plateau-2D domain, the improved version does effect a significant reduction in the magnitude of the mass balance errors. Thus, the horizontal resolution study is useful, in the sense that it confirms that the improved wetting and drying algorithm is better.

3.5. Vertical Resolution

To study the effect of vertical resolution, the number of vertical layers was varied from six layers to 201 layers for a horizontal resolution of 250 meters. Figure 8 shows the mass balance errors over the first four tidal cycles for that range of vertical resolutions. Note that, although the improved algorithm was stable for every vertical resolution in

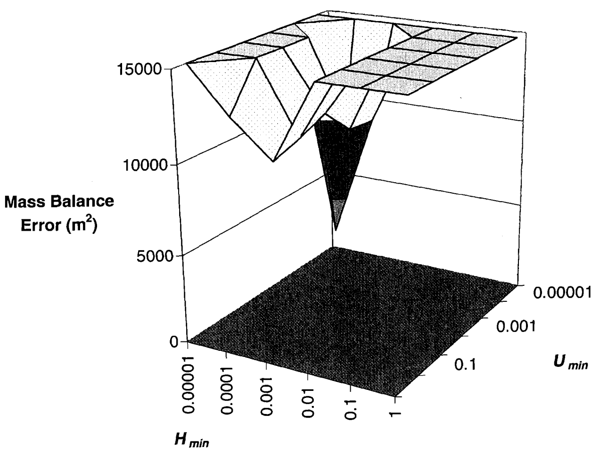

Figure 5. Mass balance errors for a range of H_{min}-U_{min} combinations, for the original wetting and drying algorithm. Note that most combinations cause the model to be unstable.

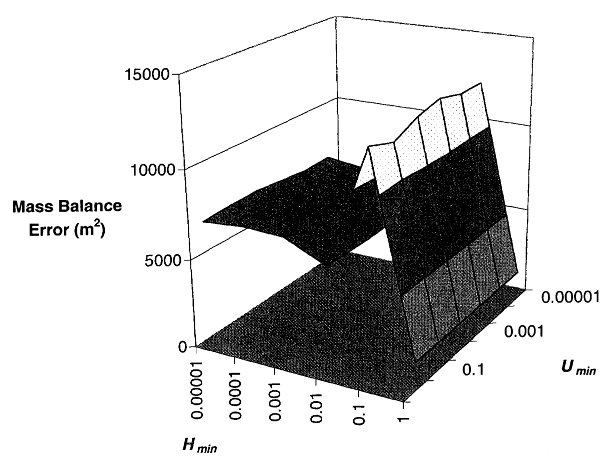

Figure 6. Mass balance errors for a range of H_{min}-U_{min} combinations, for the improved wetting and drying algorithm.

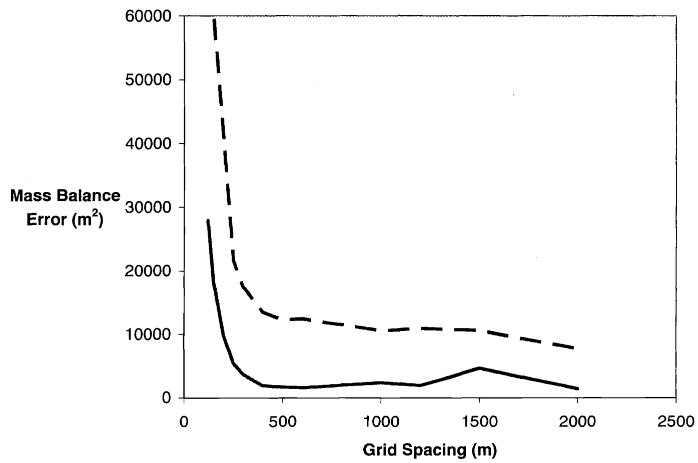

Figure 7. Mass balance errors over the four tidal cycles for the original (dashed line) and improved (solid line) wetting and drying algorithms. The improved algorithm produces mass balance errors that are considerably smaller than those from the original algorithm.

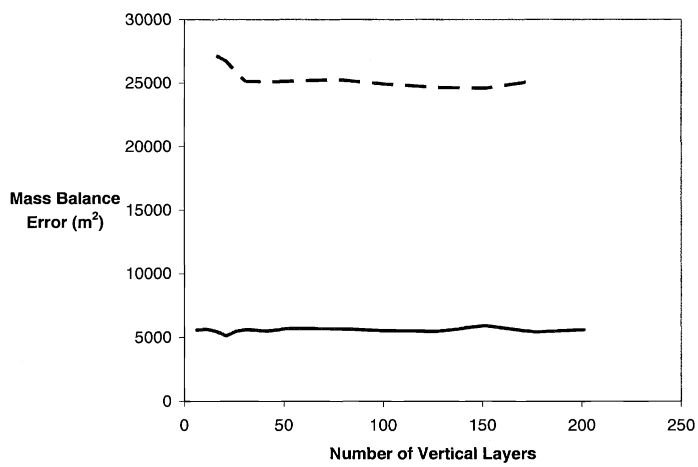

Figure 8. Mass balance errors over the first four tidal cycles for a range of vertical resolutions, for the original (dashed line) and improved (solid line) wetting and drying algorithms. The improved algorithm reduces the mass balance errors by about 80 percent.

that range, the original algorithm was unstable at both the coarse end (six to 11 layers) and the fine end (201 layers). Also note the difference in magnitude between the mass balance errors from the two algorithms. The original algorithm produces errors of about 25,000 m^2, while the improved algorithm produces errors of about 5,000 m^2. That is a decrease of about 80 percent. The improved algorithm continues to behave significantly better.

These results show that the mass balance errors are not sensitive to vertical resolution. As the number of layers is increased, the errors do not converge toward zero. Because this simulation is forced with a tide at the open ocean boundary, the flow is unidirectional throughout the water column. There is not enough vertical gradients to require additional resolution in the vertical direction, and thus the two-dimensional (*x-z*) ADCIRC model is able to simulate the problem with only a few vertical layers. In the future, a vertical resolution study should be performed on a problem that is guaranteed to have vertical gradients, such as a wind-driven or density-driven problem. However, such a study would not affect the main purpose of this paper, which is to determine the efficacy of the improvements to the wetting and drying algorithm.

4. Numerical Experiments, 3D ADCIRC

As discussed in Section 2.3, the Plateau-3D domain was designed to test the improved wetting and drying algorithm. In fact, it is a more difficult problem than the Plateau-2D domain because: (1) the flat region is longer, and (2) the sloped regions are steeper. This section contains the results of five experiments: heuristic stability, parameter sensitivity with G and K_{slip}, parameter sensitivity with H_{min} and U_{min}, horizontal resolution, and vertical resolution.

4.1. Heuristic Stability

For the Plateau-3D domain, we determined the maximum stable time step by gradually increasing the time step in increments of 5 seconds until the model became unstable. We performed this test for both the original and improved versions of the wetting and drying algorithm, in order to examine their behavior in a three-dimensional setting. The results of these tests are summarized in Table 2. The Plateau-3D domain produces different (and better) results when the improved wetting and drying algorithm is used. The time step is increased from 35 seconds to 45 seconds (or by 28 percent), and the mass balance error is roughly the same. For the rest of the numerical experiments using the three-dimensional ADCIRC model, we will only present results from the improved wetting and drying algorithm.

4.2. Parameter Sensitivity - K_{slip} and G

Here, we examine the effects of the numerical parameter G and the roughness parameter K_{slip} in a model problem. This time, it is the Plateau-3D domain. Figure 9 shows the mass balance errors per unit width of the domain, for a range of 36 combinations of these parameters. As noted in Section 2.4, the mass balance errors in

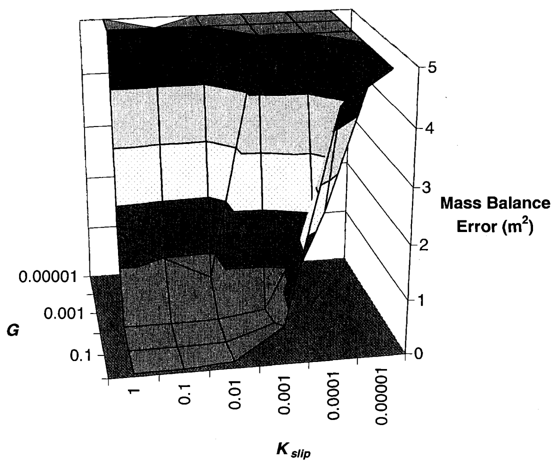

Figure 9. Mass balance errors per unit width of the Plateau-3D domain, for a range of G-K_{slip} combinations. The model is unstable in this domain for combinations when $K_{slip} = 0.00001$ and/or when $G \leq 0.0001$.

Table 2: Summary of heuristic stability results for the two versions of the wetting and drying algorithm. Note that mass balance error is an average error over all four tidal cycles.

	Original Algorithm	Improved Algorithm
Maximum stable time step	35 sec	45 sec
Mass balance error, at the maximum stable time step	105.884 m^2	112.238 m^2
Mass balance error, at a time step of 1 second	2.140 m^2	1.918 m^2

the study of the three-dimensional model are smaller because they are computed in a different manner. Note that the model is unstable in this domain when either $K_{slip} = 0.00001$ or when $G \leq 0.0001$. Also note that, except for the region in the front of Figure 9 where both parameters are large, the improved algorithm is largely unstable in this domain. We believe this behavior is due to the flood surge in the flat region and the thin film of draining water at the edge of the plateau, both of which are problem areas that were included explicitly in the design of this domain. The optimal

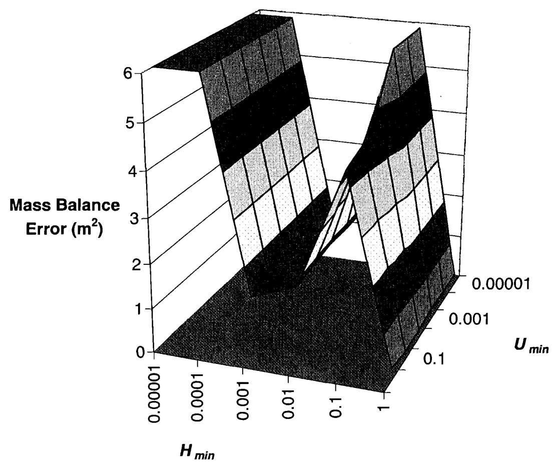

Figure 10. Mass balance errors per unit width of the Plateau-3D domain, for a range of H_{min}-U_{min} combinations. Note that the model is unstable in the Plateau-3D domain when $H_{min} < 0.001$ meters.

combination of $K_{slip} = 0.0001$ and $G = 0.01$, from both the Plateau-2D domain and from other studies using a linear sloping beach domain (Dietrich et al. 2004), lies at the edge of the optimal area in Figure 9. However, because the wetting and drying processes are damped as the bottom stress is increased, the regions where K_{slip} are maximized do not represent physically realistic simulations. These parameter sensitivity results indicate that ADCIRC can obtain acceptable mass balance properties even in adverse conditions, provided users select an appropriate combination of run-time parameters; i.e., some physical friction and an appropriate theoretically based choice of G.

4.3. Parameter Sensitivity - H_{min} and U_{min}

The drying parameter H_{min} and the wetting parameter U_{min} were varied from 10^{-5} to 10^{0} (meters for H_{min}; m/sec for U_{min}). Figure 10 shows the mass balance errors per unit width of the domain, for the range of H_{min}-U_{min} combinations. Note that the model is unstable in the Plateau-3D domain when $H_{min} < 0.001$ meters. Also note that, for the first time in any of our wetting and drying studies, we see some significant differences in the error measure when U_{min} is varied. Specifically, along the slice in the figure when $H_{min} = 0.1$ meters, the mass balance errors range from 4.3 m^2 to 5.1 m^2, and then the model becomes unstable for combinations when $U_{min} < 0.01$ m/sec.

The qualitative behavior of these results is similar to the behavior of the mass balance errors for the Plateau-2D domain shown in Figure 6. The optimal mass balance errors occur at the extremes of the stable region; however, when $H_{min} = 1.0$,

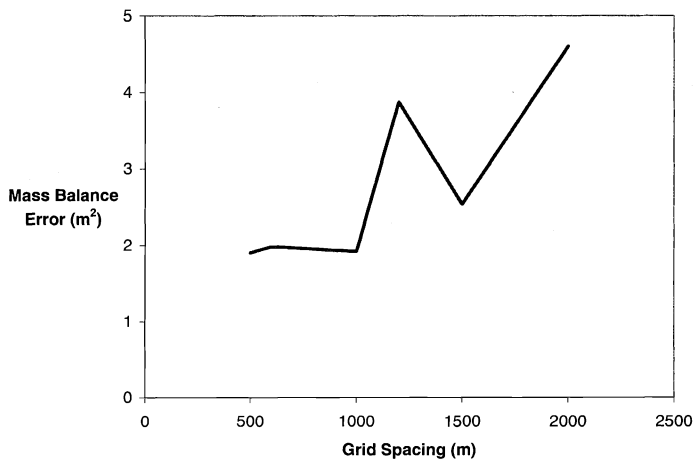

Figure 11. Mass balance errors per unit width of the Plateau-3D domain, for a range of horizontal resolutions. Note that the model was unstable in this domain for grid spacings smaller than 500 meters.

the mass balance errors are small because very little wetting and drying is allowed. The larger errors in the center of the stable region reflect the transition between H_{min} values that are not restrictive and H_{min} values that are too restrictive. In fact, when $H_{min} = 0.1$ and $U_{min} \leq 0.001$, the model becomes unstable.

4.4. Horizontal Resolution

To examine the effects of horizontal resolution on the three-dimensional wetting and drying algorithm, the grid spacing was varied from 100 meters to 2000 meters, and the mass balance errors per unit width of the domain were again examined. Figure 11 shows the results of this study. Note that the model was unstable in this domain for grid spacings less than 500 meters, at a time step of 1 second. The qualitative behavior of the errors is good, because the errors converge as the grid spacing is refined. Note that, when the grid spacing is divided by four (from 2000 meters to 500 meters), the corresponding errors decrease by a factor of 2.4, which indicates sublinear convergence. Also, at the larger grid spacings, the mass balance errors oscillate. Each data point represents a slightly different domain; for example, it is possible to begin the flat region of the domain at $x = 10.5$ kilometers if the grid spacing is 1500 meters, but the flat region must begin at $x = 12$ kilometers if the grid spacing is 2000 meters. Similar differences are observed in the ability of each domain to pinpoint the location of the wetting front. These differences in the domains cause the simulations themselves to be different, but this is a phenomenon that is mirrored with more realistic domains; as more node points are added, the underlying bathymetry is

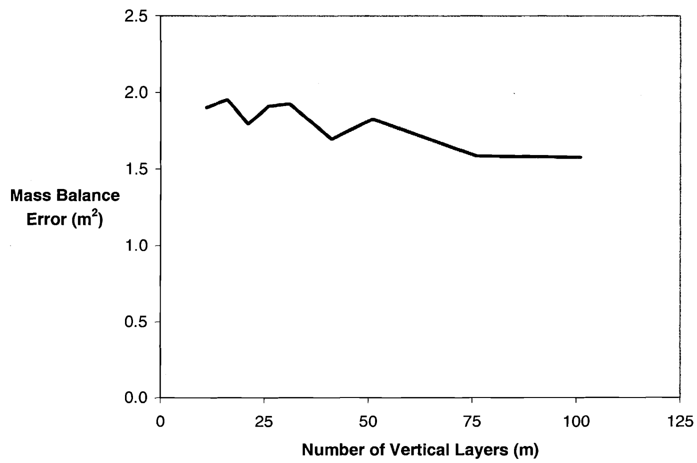

Figure 12. Mass balance errors per unit width of the domain for a range of vertical resolutions.

represented better. Thus, the oscillations and sublinear convergence are most likely a consequence of the subtle differences between the domains. Overall, we believe the observed behavior is physically realistic.

4.5. Vertical Resolution

The effect of vertical resolution was examined by varying the number of vertical layers from six to 201. Figure 12 shows the mass balance errors per unit width of the domain for a range of vertical resolutions. Note that the model was unstable at vertical resolutions with six layers and with 126 or more vertical layers. However, the results shown in the figure indicate that simulations on the Plateau-3D domain produce errors that are insensitive to vertical resolution. This finding is similar to previous studies in three dimensions (on the linear sloping beach, not shown herein) and in two dimensions (Section 3.5). Like those earlier domains, the Plateau-3D domain does not experience enough vertical variability for it to be sensitive to vertical resolution. Future studies should include a wetting and drying problem where vertical variability is guaranteed.

5. Conclusions

This study of the two-dimensional (*x-z*) ADCIRC model examined two related questions: (1) were the recent updates to the wetting and drying algorithm beneficial?, and (2) is there an optimal set of input parameters for the improved wetting and drying algorithm, for both the two-dimensional (*x-z*) and three-dimensional versions of the ADCIRC model?

The recent updates to the algorithm were beneficial. We have shown that the improved algorithm exhibits better stability or mass balance properties, and usually both. This finding is especially true with respect to the Plateau domain, which was designed specifically to test the improved algorithm under the conditions for which it was implemented. In the Plateau-2D domain, the improved algorithm doubled the maximum stable time step, proved stable for a wider range of parameters than did the original algorithm, and produced significantly less mass balance errors. In the Plateau-3D domain, the improved algorithm increased the maximum stable time step by 28 percent and allowed for simulation of a relatively difficult model problem. The updates to the wetting and drying algorithm are worthwhile.

And, when the improved algorithm is used, there are obvious consequences for the selection of input parameters. The Plateau domain behaves well at the combination of $K_{slip} = 0.0001$ and $G = 0.01$ sec^{-1}; however, users of the model should select values for these parameters that are consistent with the physics of the problem. The H_{min} parameter produces the best behavior when it is set to a relatively low value, such as $H_{min} \leq 0.01$ meters. The U_{min} parameter does not have an effect on model behavior. Results for the horizontal resolution studies were mixed, although qualitative convergence was observed with the three-dimensional ADCIRC model. And, for vertical resolution, results confirmed that our problems did not involve sufficient vertical variability.

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