Performance of the Integrally-Coupled, Unstructured-Mesh SWAN+ADCIRC(DG) Model

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Tuesday, 22 March 2011

SWAN+ADCIRC:
'Tight' Coupling of Hurricane Waves and Surge

M. Zijlema (2010). "Computation of Wind-Wave Spectra in Coastal Waters with SWAN on Unstructured Grids." *Coastal Engineering*, 57, 267-277.

J.C. Dietrich, *et al*. (2011). "Modeling Hurricane Waves and Storm Surge using Integrally-Coupled, Scalable Computations." *Coastal Engineering*, 58, 45-65.

J.C. Dietrich, *et al*. (2011). "Hurricane Gustav (2008) Waves and Storm Surge: Hindcast, Synoptic Analysis and Validation in Southern Louisiana." *Monthly Weather Review*, in press.

A.B. Kennedy, *et al.* (2011). "Origin of the Hurricane Ike Forerunner Surge." *Geophysical Research Letters*, in press.

J.C. Dietrich, *et al.* (2011). "Performance of the Unstructured-Mesh, SWAN+ADCIRC Model in Computing Hurricane Waves and Surge." *Journal of Scientific Computing*, in preparation.

SWAN : Simulating WAves Nearshore

Governing Equation:

- Conserves action density $N = N(t, x, y, \theta, \sigma)$:

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}\left[\left(c_x + U\right)N\right] + \frac{\partial}{\partial y}\left[\left(c_y + V\right)N\right] + \frac{\partial}{\partial \theta}\left[c_\theta N\right] + \frac{\partial}{\partial \sigma}\left[c_\sigma N\right] = \frac{S_{tot}}{\sigma}$$

where: t is time;

x and y are the geographic directions;

c_x and c_y are the propagation velocities in geographic space;

U and V are the components of the currents;

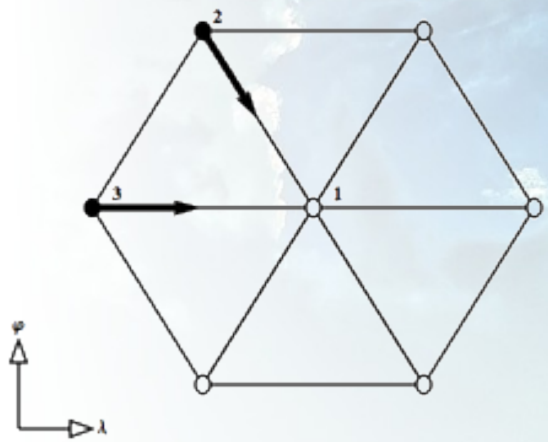
c_θ is the propagation velocity in the direction θ;

c_σ is the propagation velocity in the direction σ;

and S_{tot} represents source and sink terms.

Solution Algorithm:

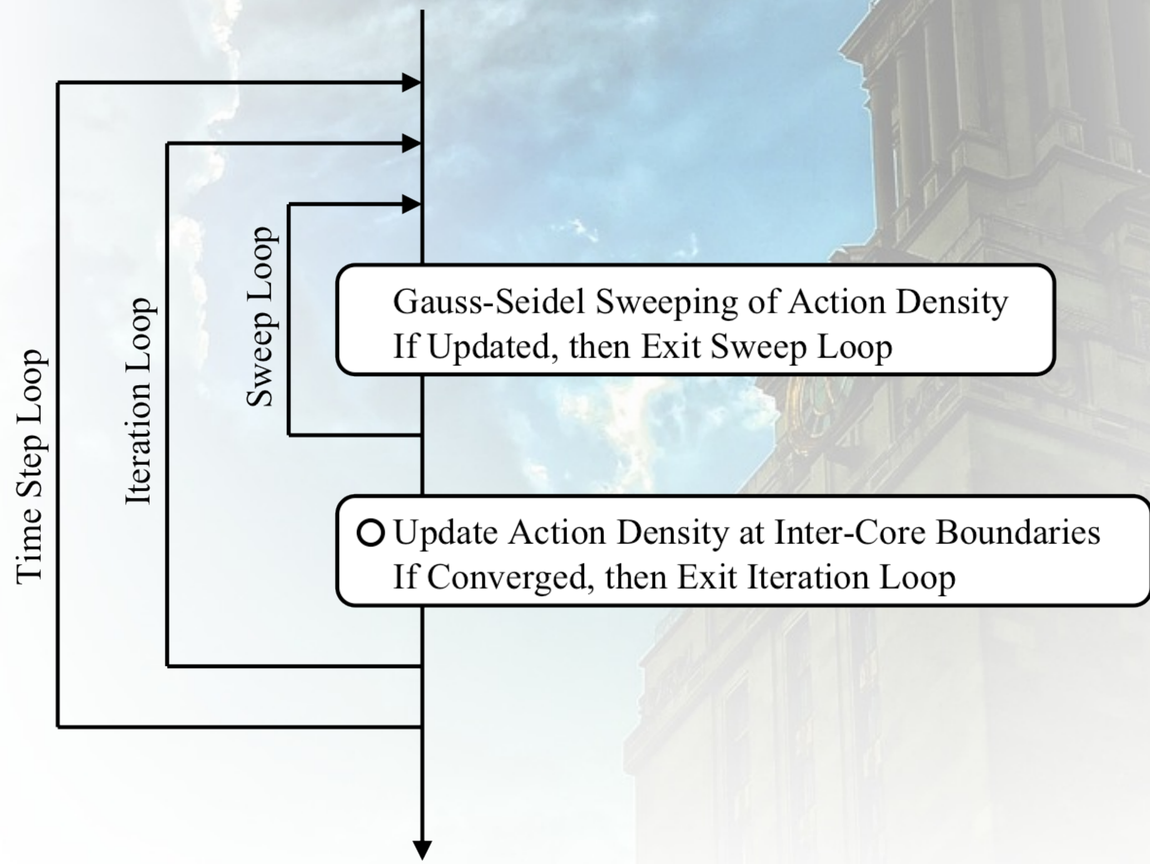
- Action density is propagated from updated vertices (2,3) to unknown vertices (1), via a mapping to a reference element:



SWAN : Simulating WAves Nearshore

Solution Algorithm:

- Propagates action density via Gauss-Seidel sweeping:



ADCIRC : ADvanced CIRCulation

Governing Equations:

- Solves the Generalized Wave Continuity Equation (GWCE):

$$\frac{\partial^2 \zeta}{\partial t^2} + \tau_0 \frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{J}_x}{\partial x} + \frac{\partial \tilde{J}_y}{\partial y} - UH \frac{\partial \tau_0}{\partial x} - VH \frac{\partial \tau_0}{\partial y} = 0$$

where:

$$\tilde{J}_x = -Q_x \frac{\partial U}{\partial x} - Q_y \frac{\partial U}{\partial y} + fQ_y - \frac{g}{2}\frac{\partial \zeta^2}{\partial x} - gH \frac{\partial}{\partial x}\left[\frac{p_s}{g\rho_0} - \alpha\eta\right] + \frac{\tau_{sx} + \tau_{bx}}{\rho_0} + \left(M_x - D_x\right) + U\frac{\partial \zeta}{\partial t} + \tau_0 Q_x - gH\frac{\partial \zeta}{\partial x}$$

$$\tilde{J}_y = -Q_x \frac{\partial V}{\partial x} - Q_y \frac{\partial V}{\partial y} - fQ_x - \frac{g}{2}\frac{\partial \zeta^2}{\partial y} - gH \frac{\partial}{\partial y}\left[\frac{p_s}{g\rho_0} - \alpha\eta\right] + \frac{\tau_{sy} + \tau_{by}}{\rho_0} + \left(M_y - D_y\right) + V\frac{\partial \zeta}{\partial t} + \tau_0 Q_y - gH\frac{\partial \zeta}{\partial y}$$

- Solves the vertically-integrated momentum equations:

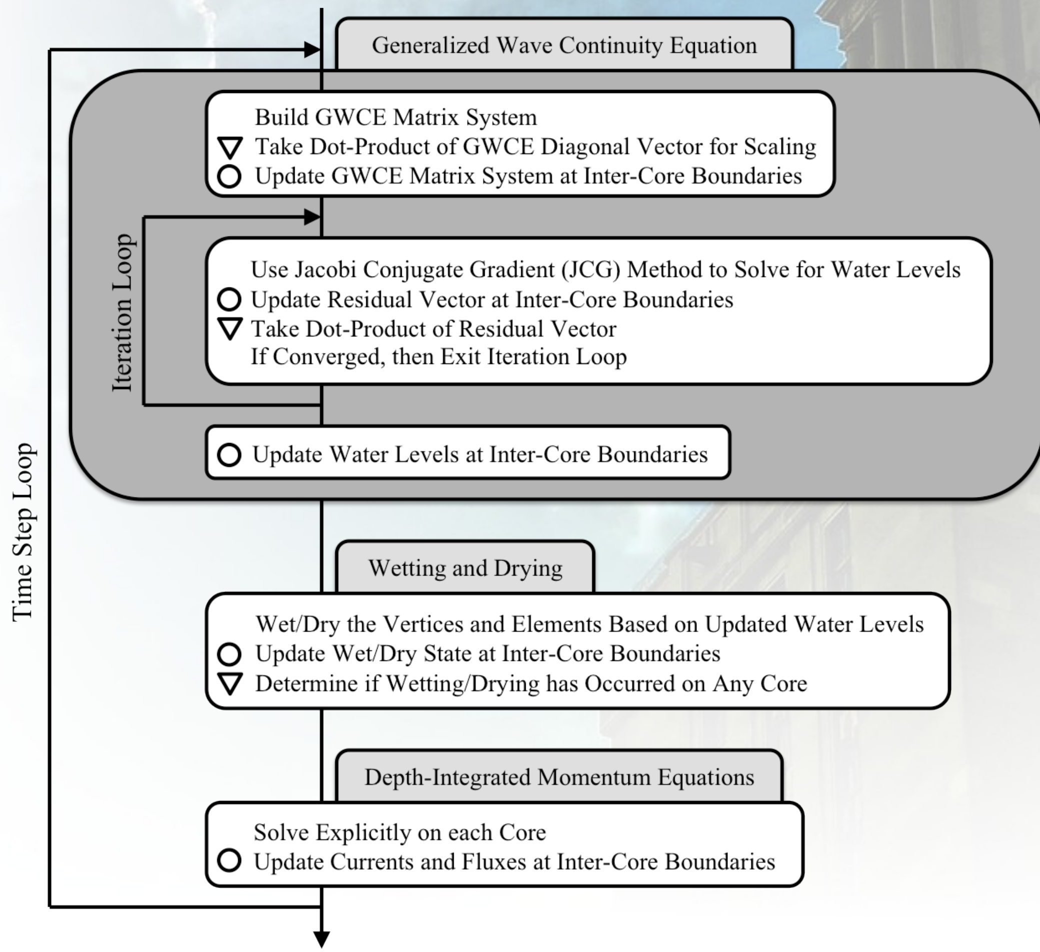
$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} - fV = -g\frac{\partial}{\partial x}\left[\zeta + \frac{p_s}{g\rho_0} - \alpha\eta\right] + \frac{\tau_{sx} + \tau_{bx}}{\rho_0 H} + \frac{M_x - D_x}{H}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + fU = -g\frac{\partial}{\partial y}\left[\zeta + \frac{p_s}{g\rho_0} - \alpha\eta\right] + \frac{\tau_{sy} + \tau_{by}}{\rho_0 H} + \frac{M_y - D_y}{H}$$

ADCIRC : ADvanced CIRCulation

Solution Algorithm:

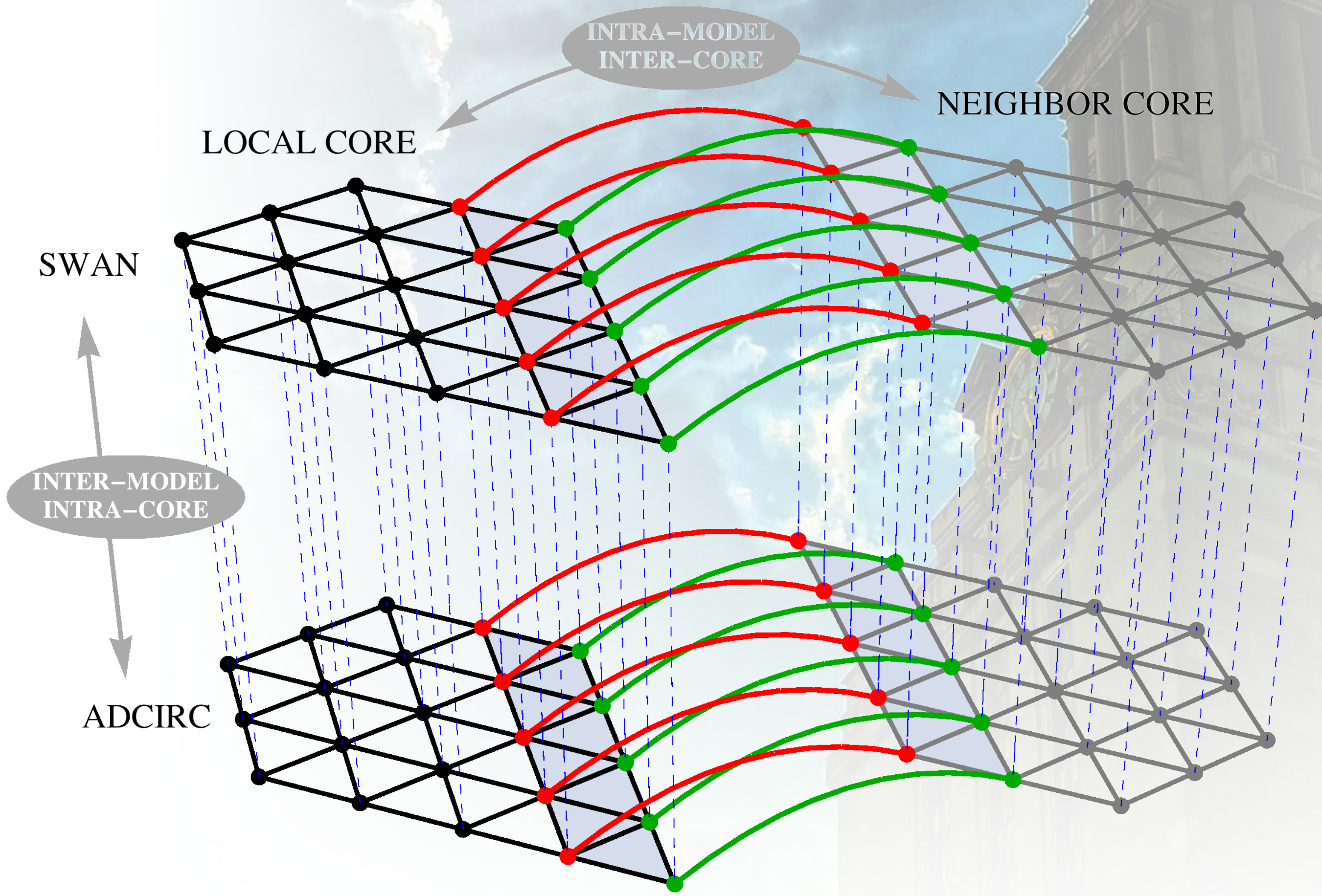
- Utilizes a Continuous Galerkin (CG) finite element discretization.
- Computes water levels and currents in three stages:



SWAN+ADCIRC

‘Tight’ Coupling:

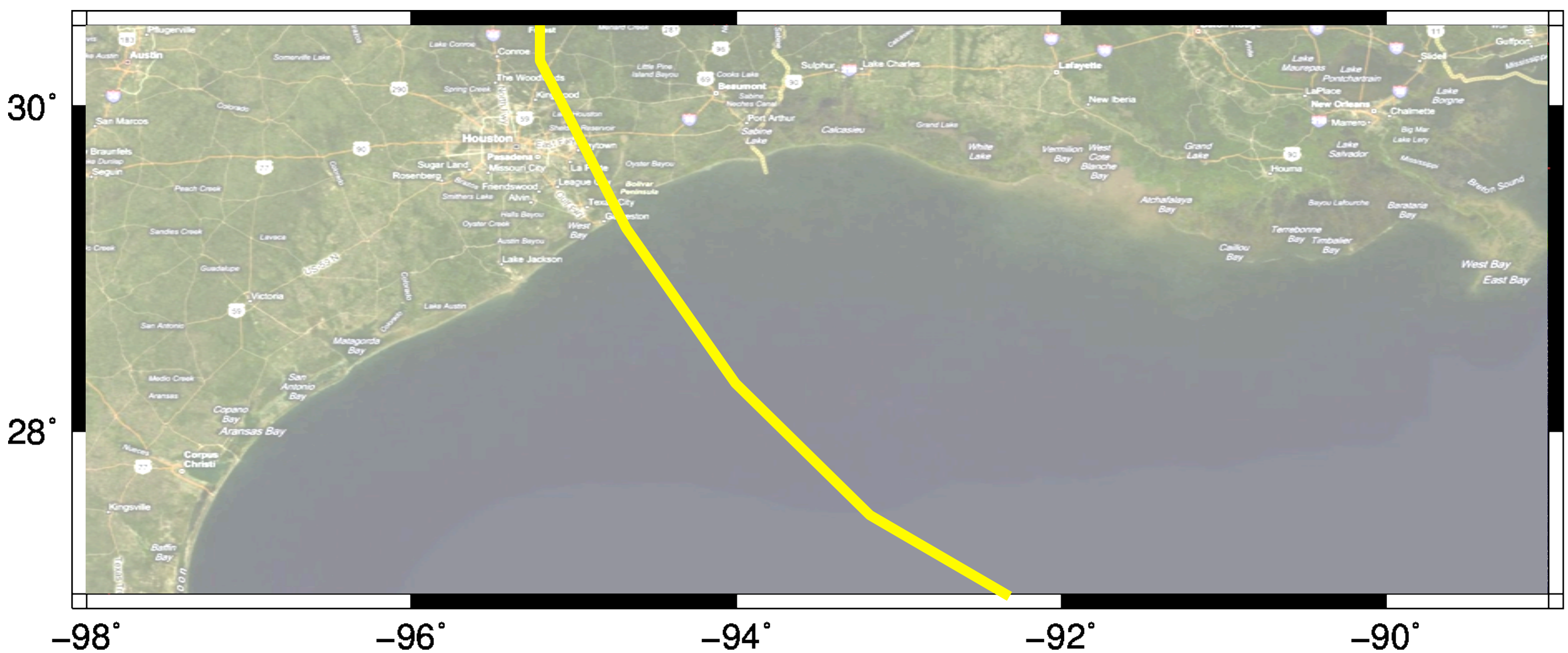
- Models share same local sub-meshes, communicate locally:



Ike on the LATEX Shelf

Hurricane Ike (2008):

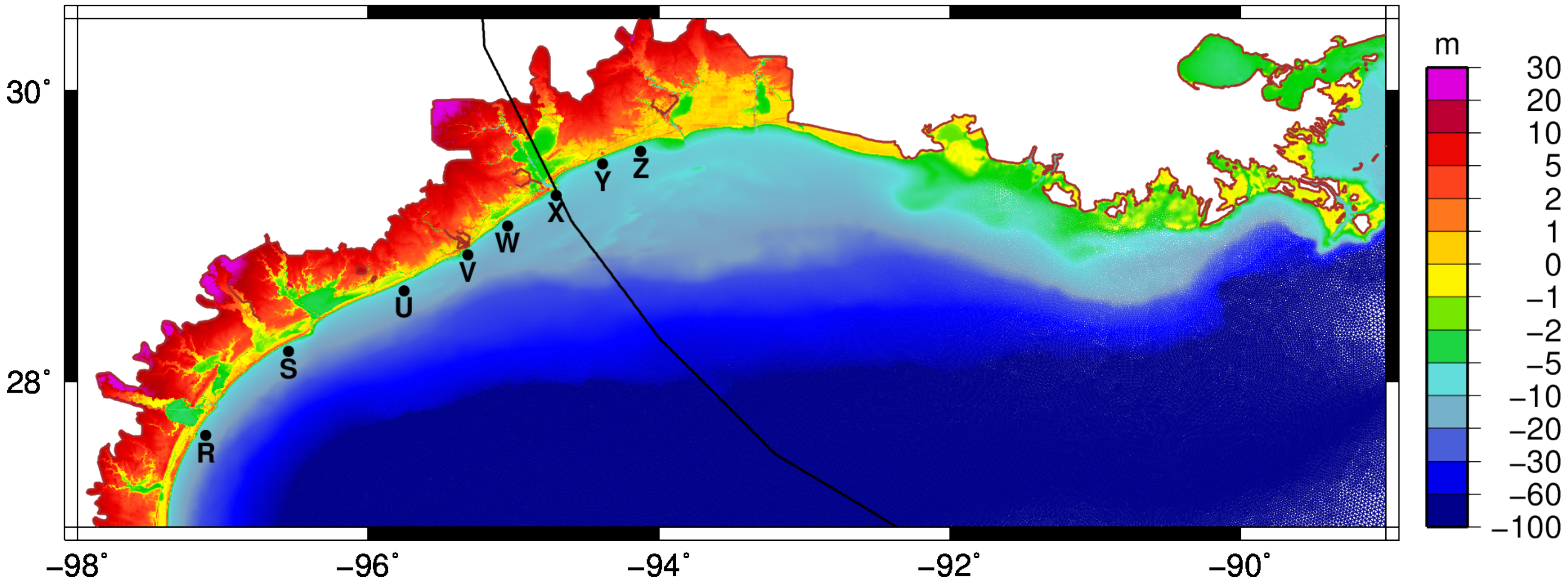
- Made landfall near Galveston, TX:



Ike on the LATEX Shelf

Texas Mesh:

- Bathymetry and Topography:



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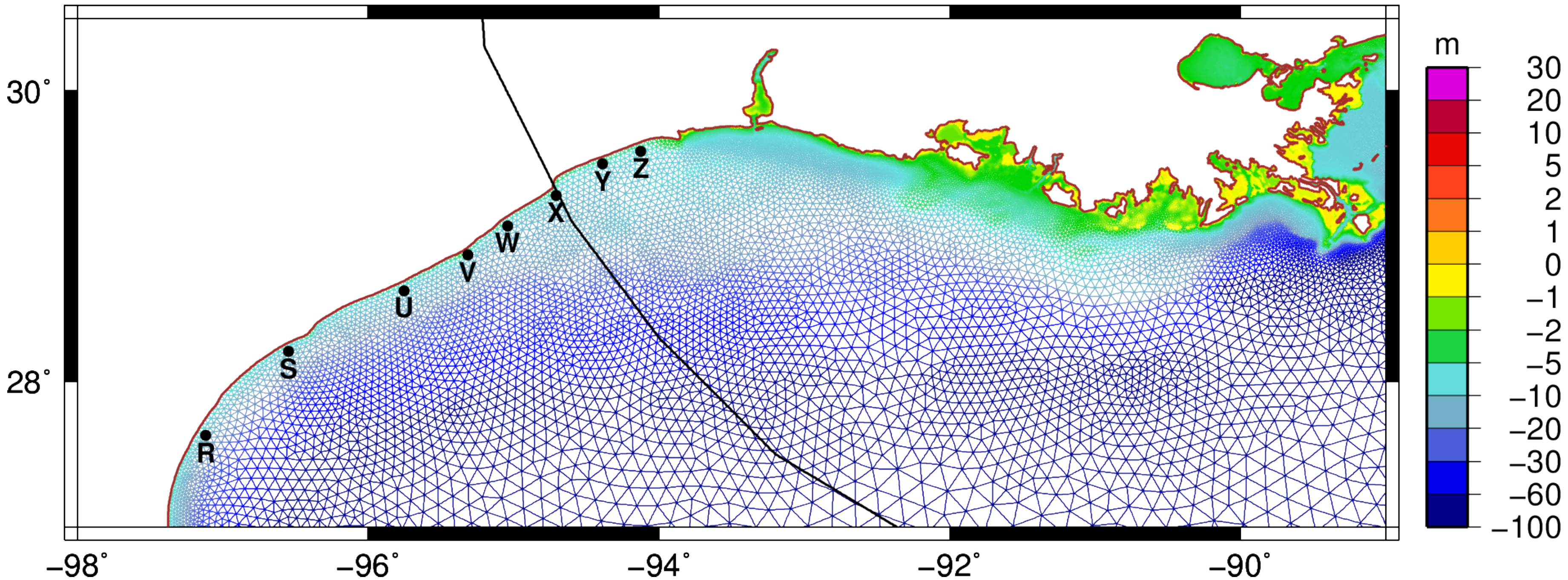
EC2001 Mesh:

- Bathymetry and Topography:



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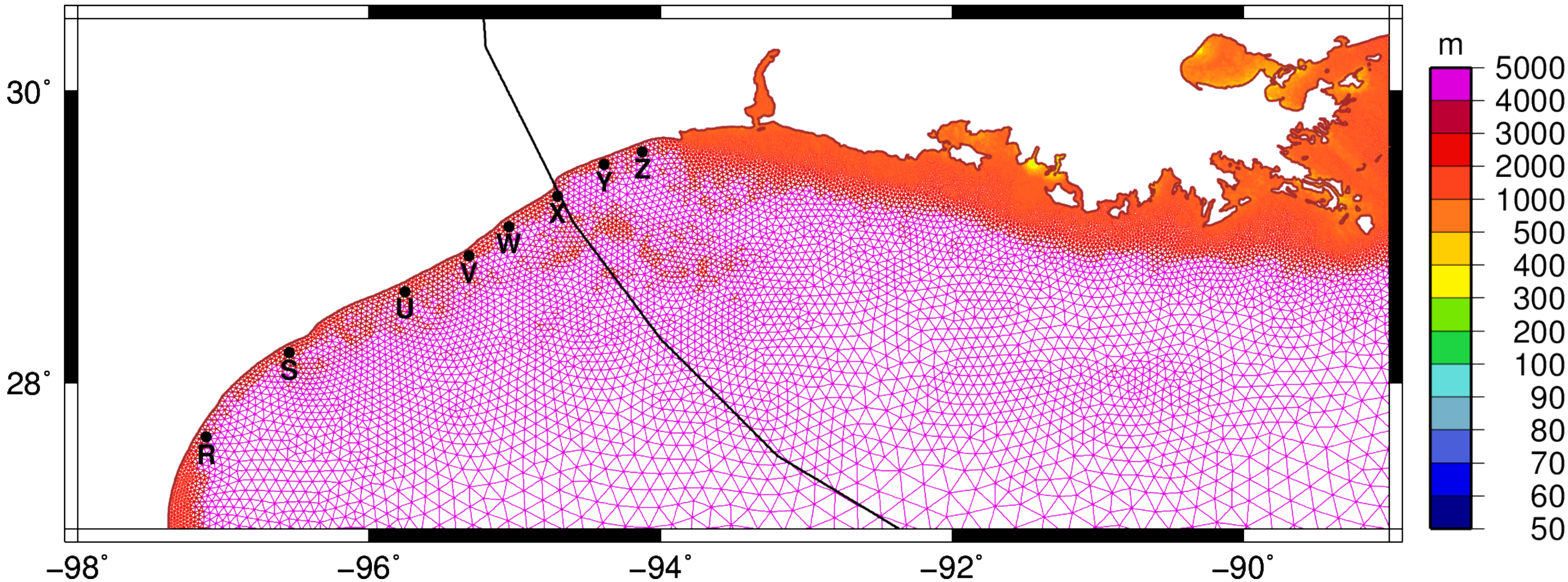
EC2001 Mesh:

- Mesh Sizes:



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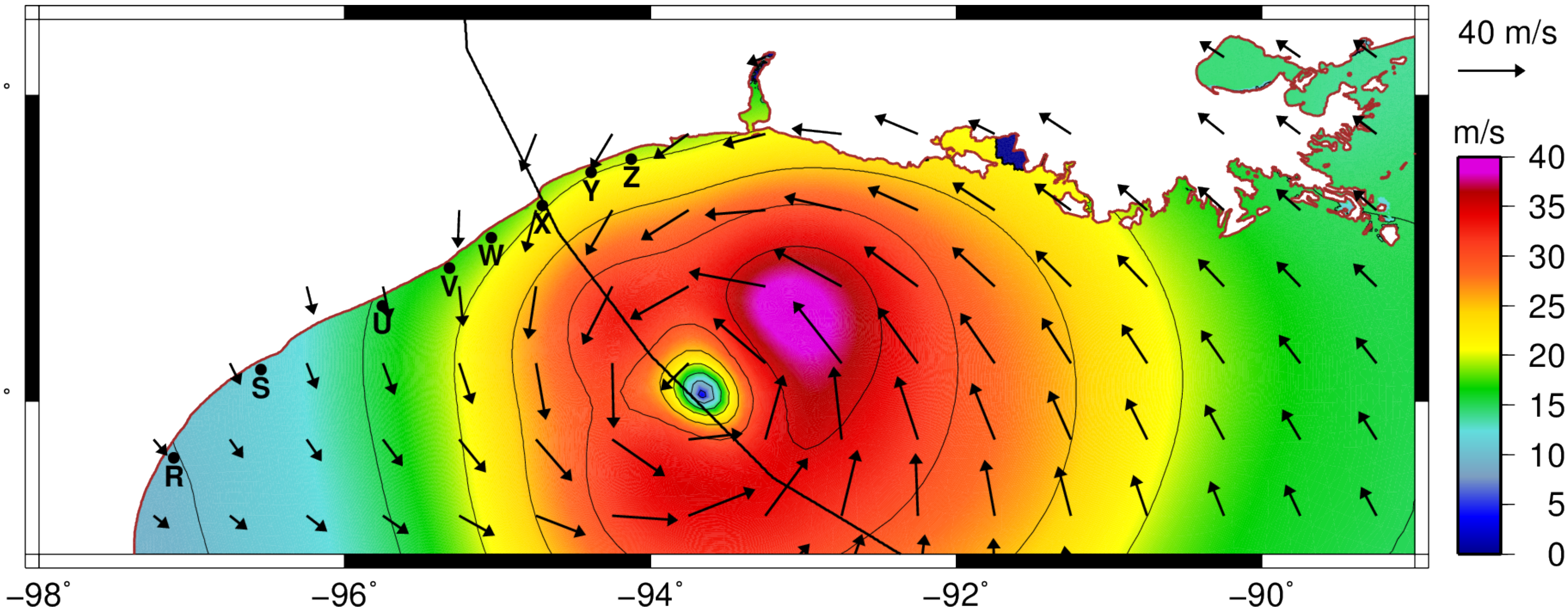
2008 / 09 / 12 / 2200Z:

- Wind Speeds:



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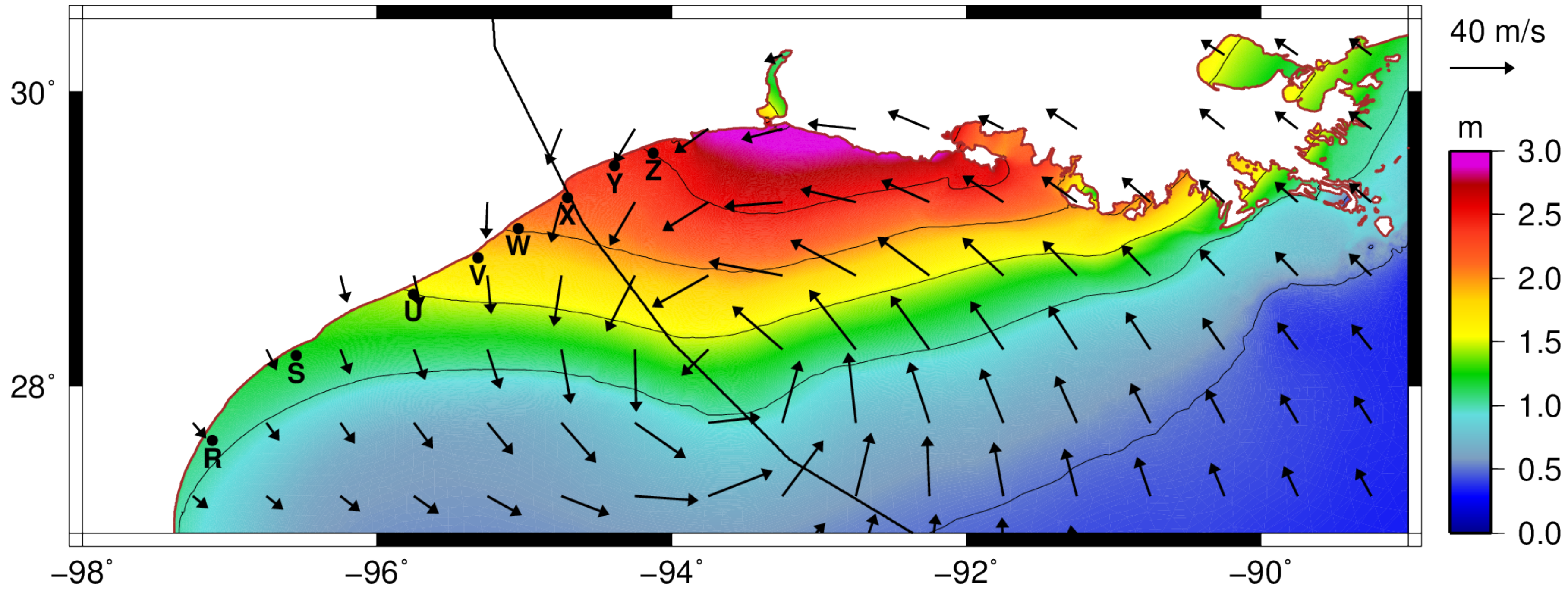
- ADCIRC Water Levels:



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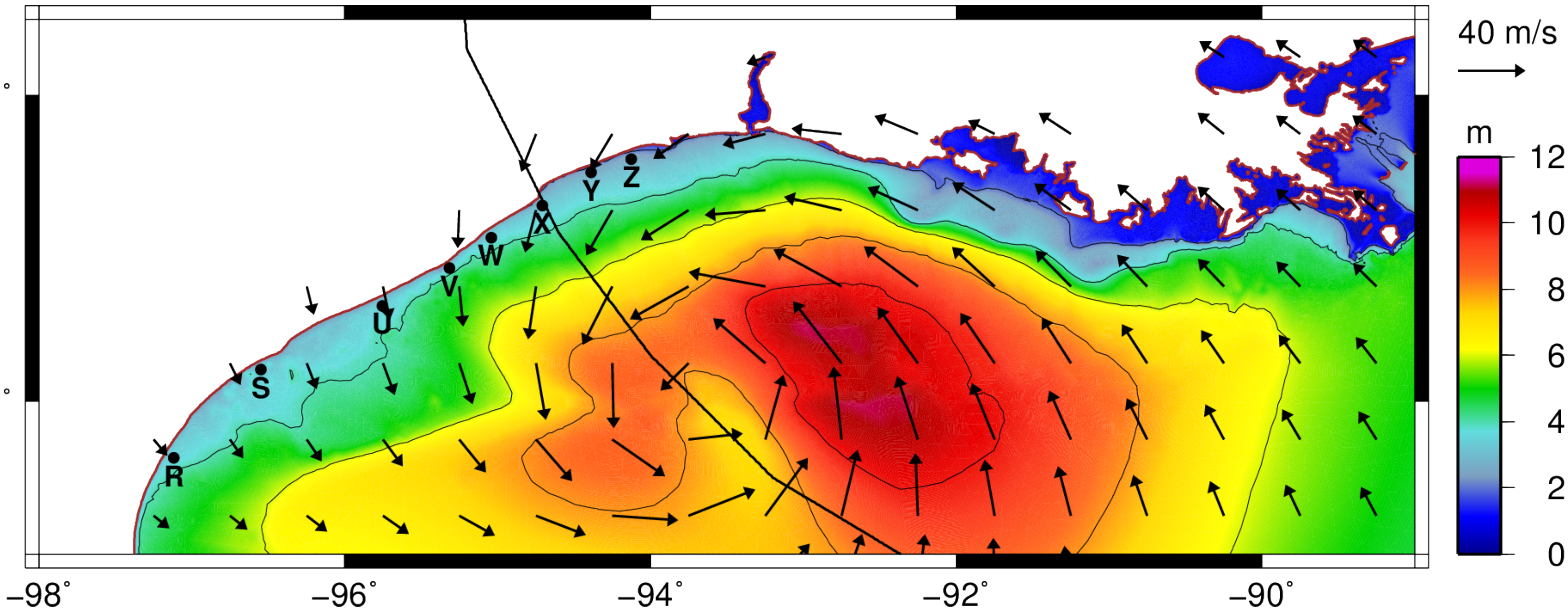
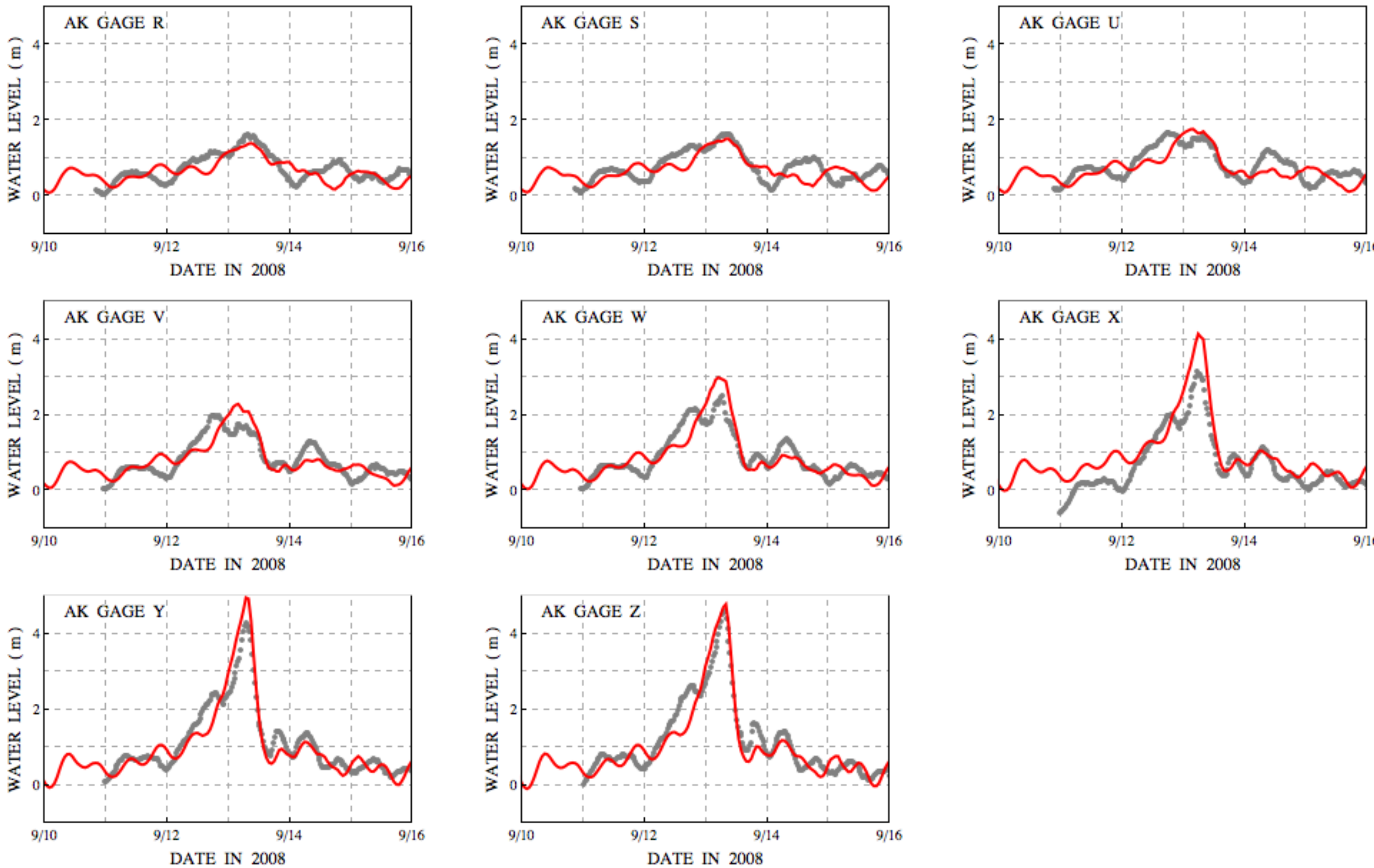
- SWAN+ADCIRC Wave Heights:



Ike on the LATEX Shelf

Kennedy Gages:

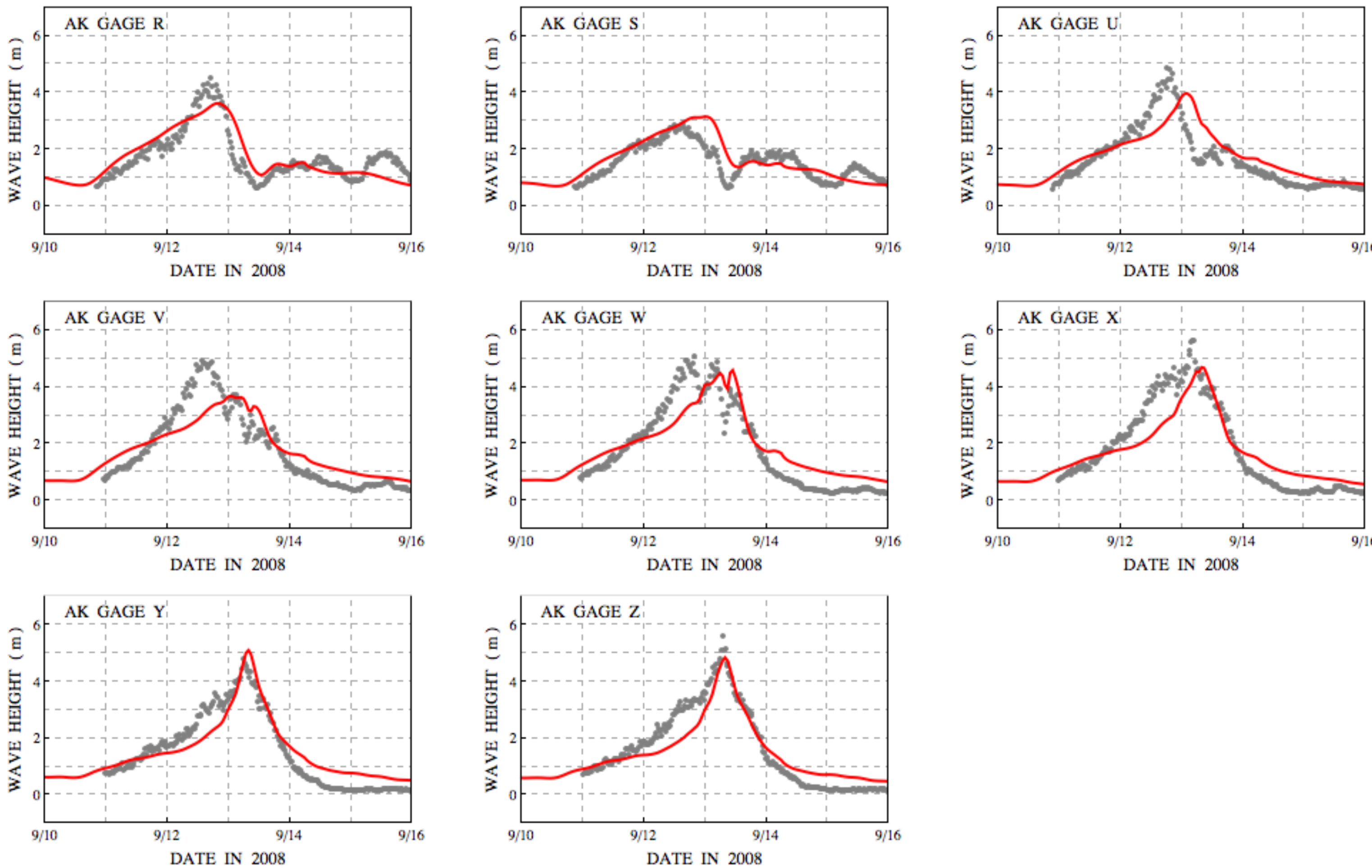
- ADCIRC Water Levels:



Ike on the LATEX Shelf

Kennedy Gages:

- SWAN+ADCIRC Wave Heights:



SWAN+DG:

Effect of Circulation Numerics on Nearshore Waves

E.J. Kubatko, *et al.* (2006). "*hp* Discontinuous Galerkin Methods for Advection Dominated Problems in Shallow Water Flow.*"* *Computer Methods in Applied Mechanics and Engineering*, 196, 437-451.

C.N. Dawson, *et al.* (2011). "Discontinuous Galerkin Methods for Modeling Hurricane Storm Surge." *Advances in Water Resources*, in press.

J.C. Dietrich, *et al.* (2011). "Effect of Coupled Circulation on a Nearshore Wave Model." *Coastal Engineering*, in preparation.

DG : Discontinuous Galerkin

Governing Equations:

- Solves primitive continuity equation for water levels:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}(UH) + \frac{\partial}{\partial y}(VH) = 0$$

- Solves momentum equations for currents:

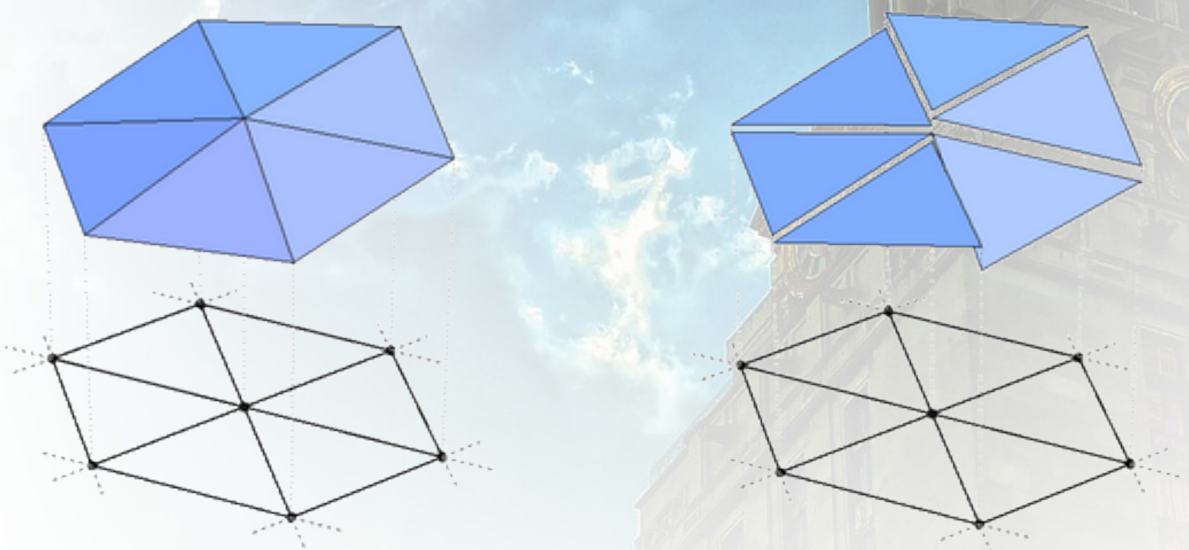
$$\frac{\partial}{\partial t}(UH) + \frac{\partial}{\partial x}(UH) + \frac{\partial}{\partial y}(UH) = fVH - H\frac{\partial}{\partial x}\left[g(\zeta - \alpha\eta) + \frac{p_s}{\rho_0}\right] + \frac{\tau_{sx}}{\rho_0} - \tau_{bx}UH + \nu_T\left[\frac{\partial^2}{\partial x^2}(UH) + \frac{\partial^2}{\partial y^2}(UH)\right]$$

$$\frac{\partial}{\partial t}(VH) + \frac{\partial}{\partial x}(VH) + \frac{\partial}{\partial y}(VH) = -fUH - H\frac{\partial}{\partial y}\left[g(\zeta - \alpha\eta) + \frac{p_s}{\rho_0}\right] + \frac{\tau_{sy}}{\rho_0} - \tau_{by}VH + \nu_T\left[\frac{\partial^2}{\partial x^2}(VH) + \frac{\partial^2}{\partial y^2}(VH)\right]$$

DG : Discontinuous Galerkin

Solution Algorithm:

- Uses local basis functions that can be *p*-adaptive.
- Solution can be discontinuous along element edges:



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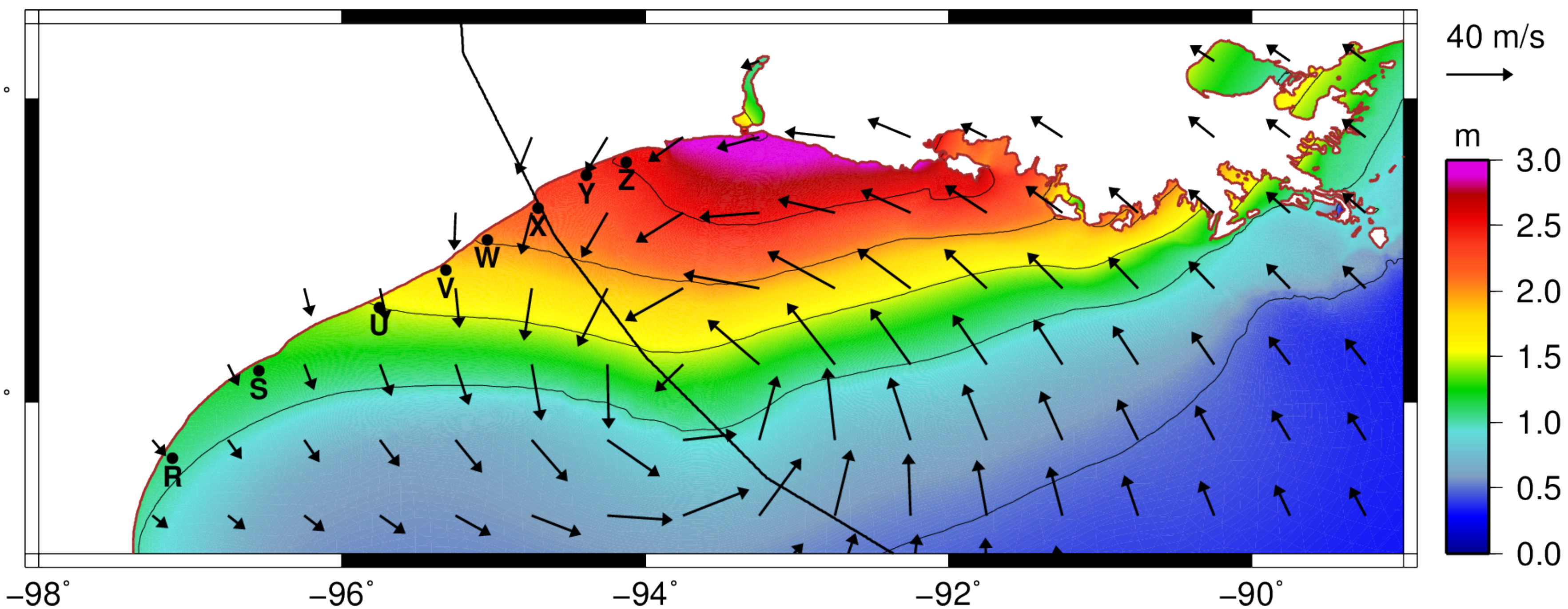
- DG Water Levels:



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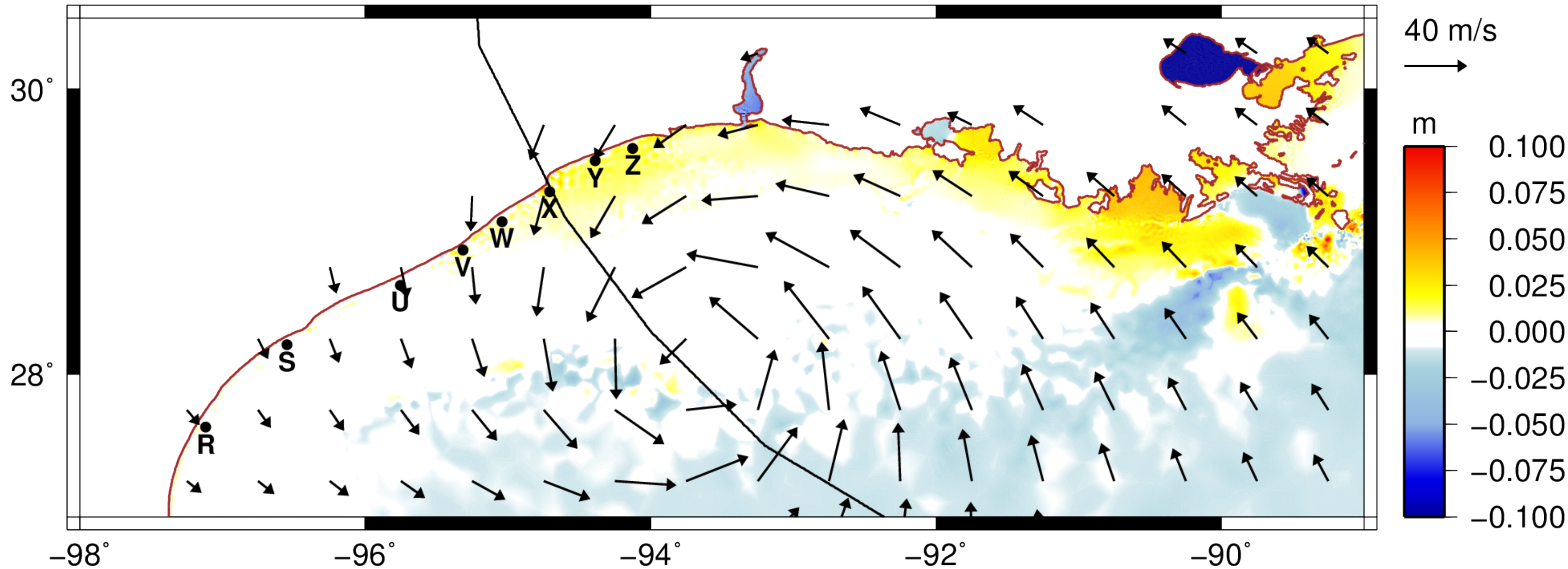
- Difference between DG and ADCIRC Water Levels:



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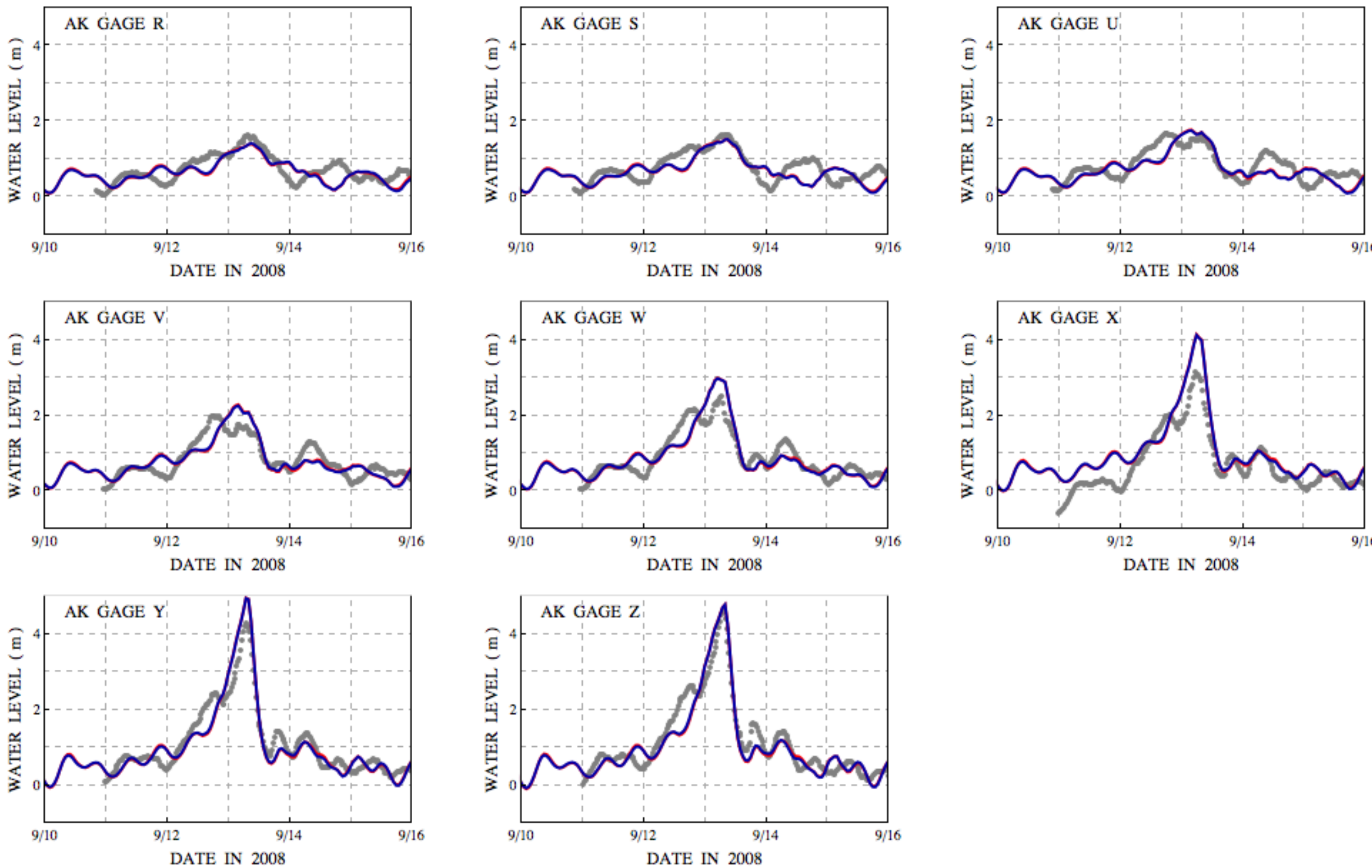
Kennedy Gages:

- DG Water Levels:



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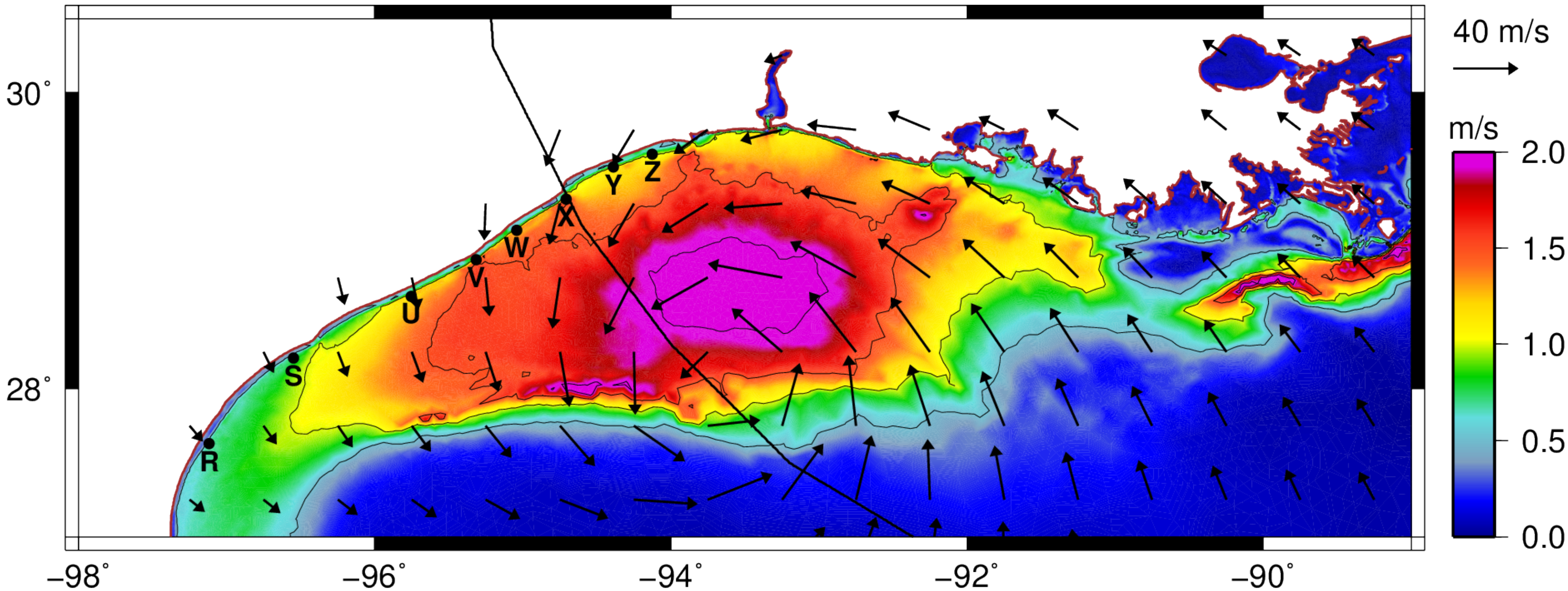
- DG Currents:



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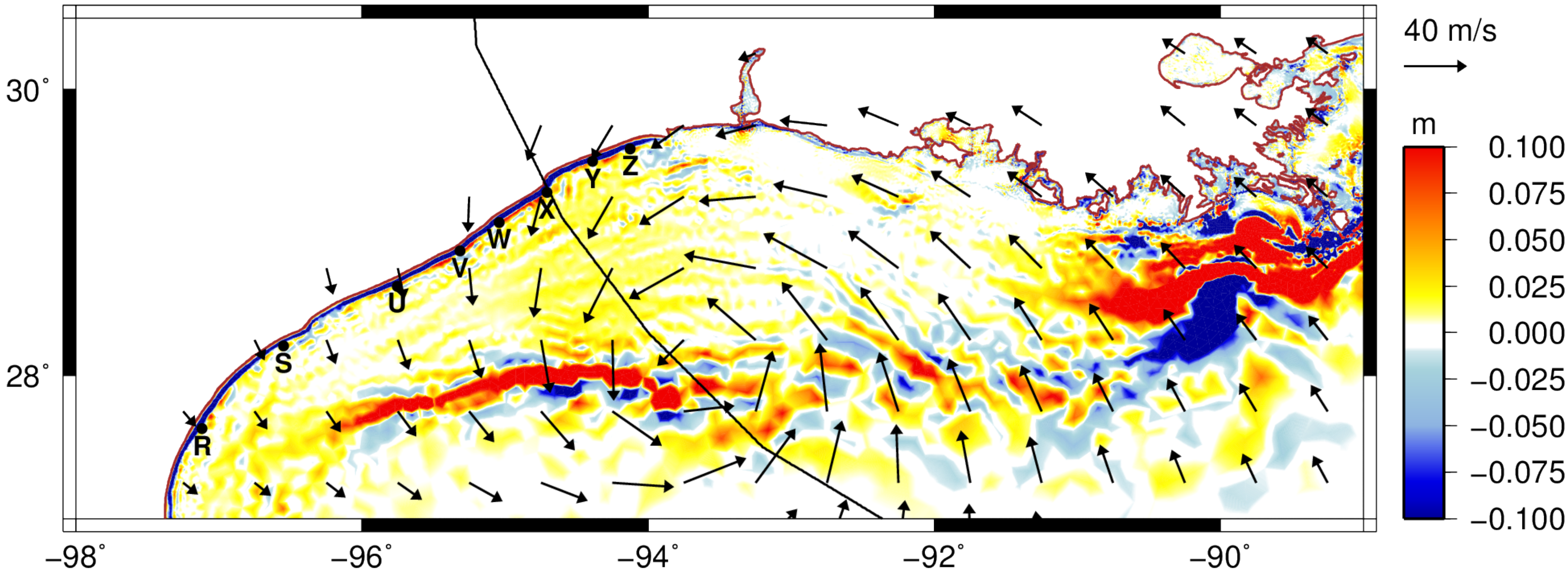
2008 / 09 / 12 / 2200Z:

- Difference between DG and ADCIRC Currents:



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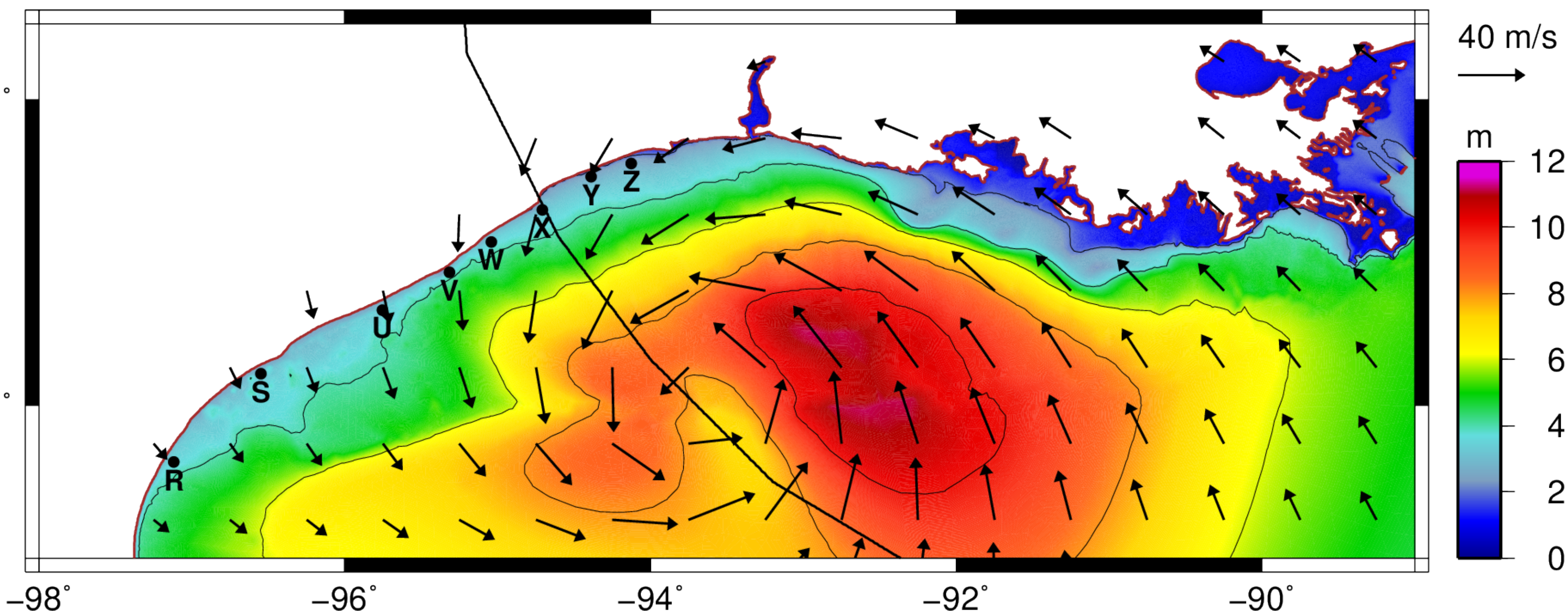
- SWAN+DG Wave Heights:



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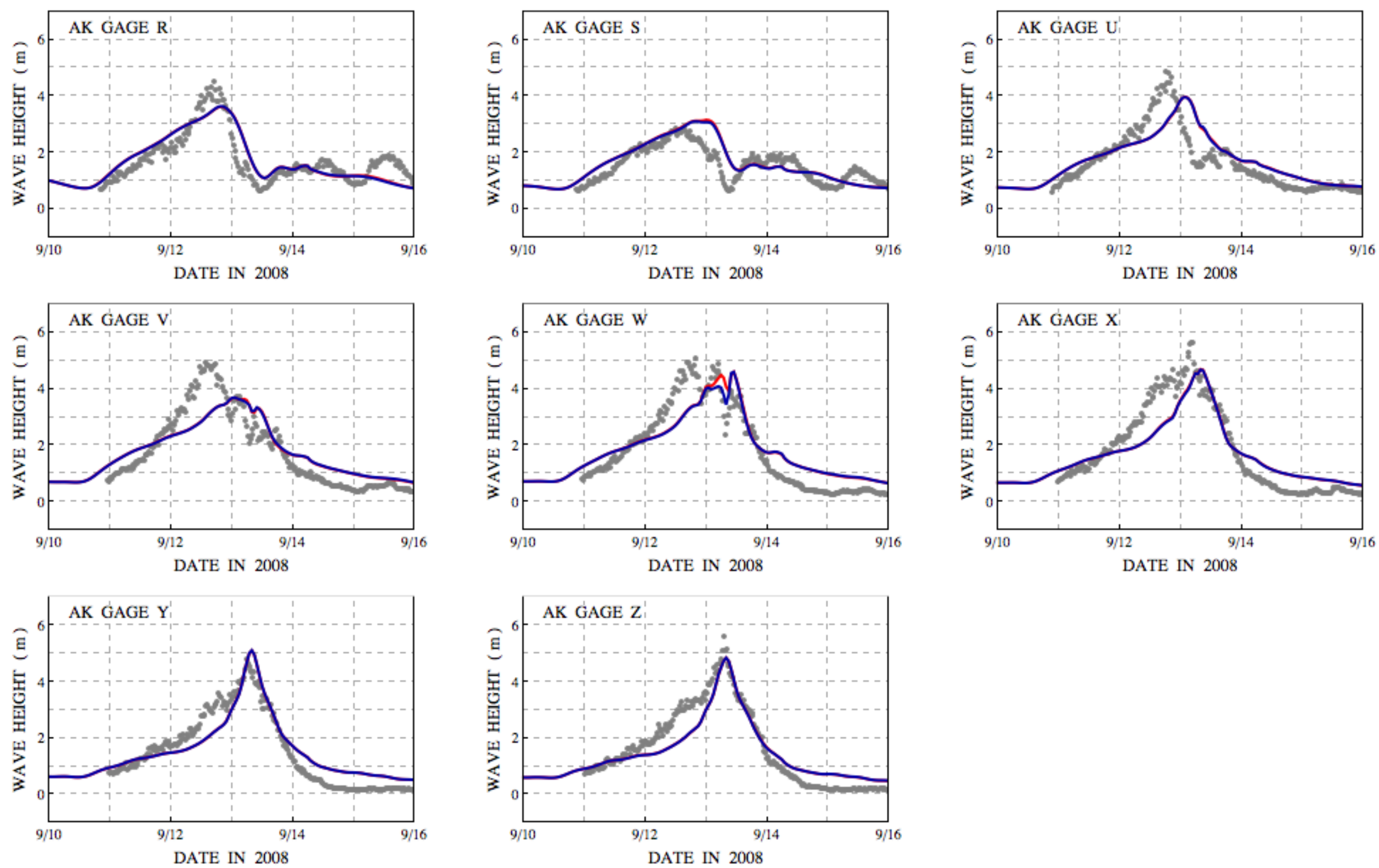
Kennedy Gages:

- SWAN+DG Wave Heights:



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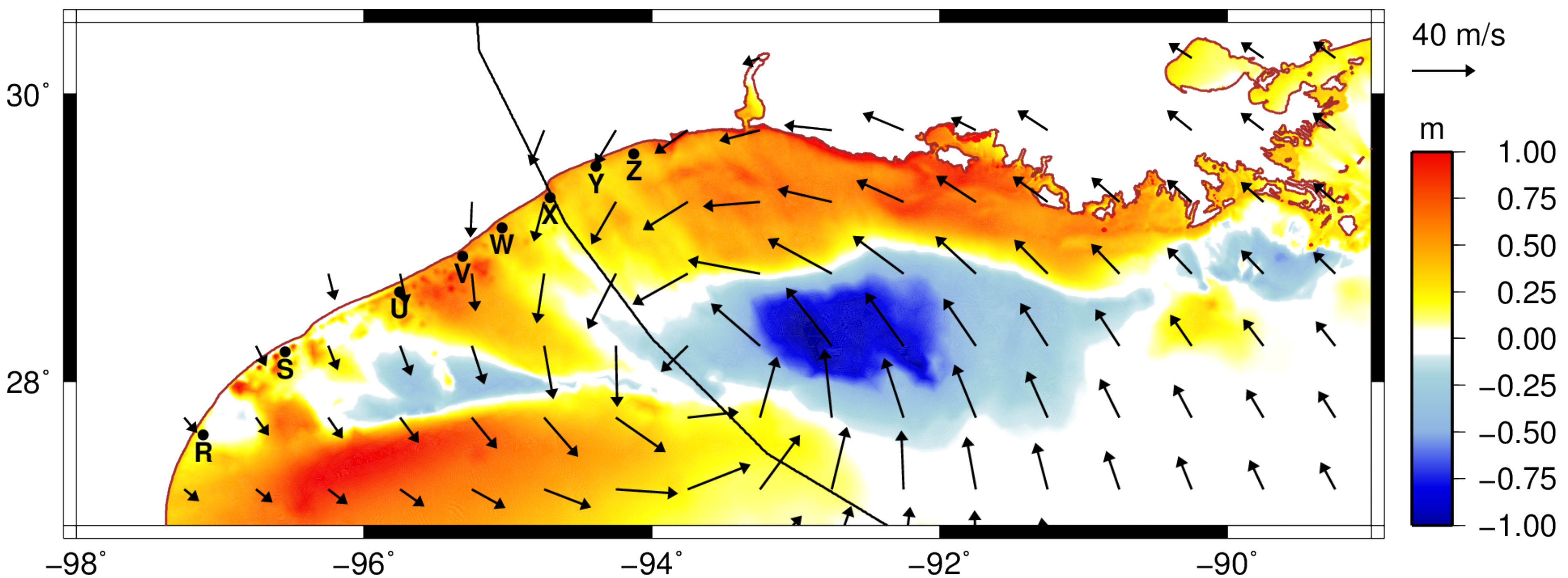
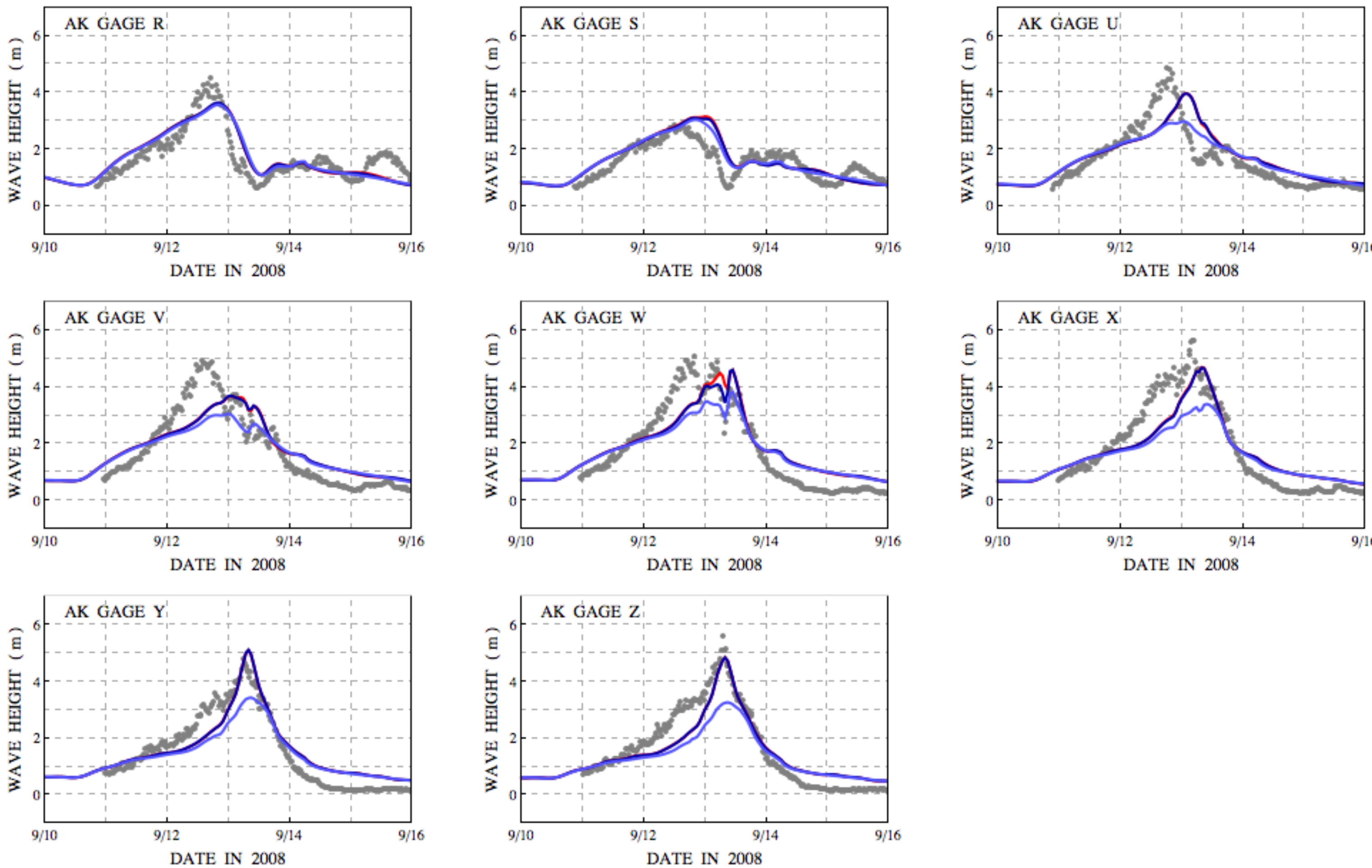
- Effect of Circulation on SWAN+DG Wave Heights:



Ike on the LATEX Shelf

Kennedy Gages:

- Effect of Circulation on SWAN+DG Wave Heights:



The End

Conclusions:

- DG model produces circulation that is very similar to ADCIRC:
    - Water levels are nearly identical.
    - Currents are more peaked in regions with bathymetric gradients.
- SWAN+DG simulates well the waves and storm surge on the shelf.
- SWAN solution is sensitive to circulation:
    - Wave heights increased by 1m.
    - Wave periods increased by 4s (but not shown herein).

Future Work:

- Extend SWAN+DG to high-resolution Texas mesh.